On-line Joint QoS Routing and Channel Assignment in Multi-Channel Multi-Radio Wireless Mesh Networks[☆]

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Abstract

We study the problem of on-line joint QoS routing and channel assignment for performance optimization in multi-channel multi-radio wireless mesh networks, which is a fundamental issue in supporting quality of service for emerging multimedia applications. To our best knowledge, this is the first time that the problem is addressed. Our proposed solution is composed of a routing algorithm that finds up to k but not necessarily feasible paths for each demand and an on-demand channel (re)assignment algorithm that adapts network resources to maintain feasibility of one of the paths. We also study the problem of obtaining an upper bound on the network performance. First, we consider an artificial version of the problem, in which all demands arrive at the same time, and formulate it as a mixed integer linear programming model. To tackle the complexity of the model, it is relaxed that provides a tight upper bound while improves solution time up to 3.0e+5 times. Then, we model the original problem by extending the relaxed model to consider dynamic demands, it leads to a huge model; thus, we develop another model, which is equivalent to the first one and is *decomposable*. It is broken down by a decomposition algorithm into subproblems, which are solved sequentially. Our extensive simulations show that the proposed solution has comparable performance to the bound obtained from the decomposition algorithm; it efficiently exploits available channels, and needs very few radios per node to achieve high network performance.

Keywords: Joint QoS Routing and Channel Assignment, Optimization Model, Decomposition, Upper Bound, Multi-Channel Multi-Radio Wireless Mesh Networks

1. Introduction

QoS of Service (QoS) support, which is entailed by emerging multimedia services, is an essential component in broadband Wireless Mesh Networks (WMN).

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It is challenging since multimedia services require intensive resources and the capacity of WMNs is shrunk by the interferences arise from the shared nature of the wireless media. Multi-channel multi-radio networking is a promising approach to mitigate the interferences and boost network capacity.

The main problem is to maximize network performance while maintaining QoS requirements. Contrary to the traditional network throughput maximization problem, in this problem, the network performance is measured in terms of acceptance rate of *QoS sensitive traffic demands*. A demand is accepted if the network can meet its QoS requirements. Due to the fact that bandwidth is the most important QoS requirement for multimedia applications, which influences other requirements such as delay jitter as well [1], we focus on this requirement. Consequently, in the problem studied in this paper, a demand is accepted if there is a path with sufficient bandwidth that is named *feasible path*.

Existence of the feasible path depends on available bandwidth of links, which is specified by channel assignment pattern and flow routes. It depends on channel assignment because each link has to share its physical channel capacity with other interfering links, which are determined by the channel assignment. Flow routing affects links available bandwidth as it specifies the load on each link. Therefore, to maximize the network performance, routing path of flows and channels of links should be jointly optimized that leads to the *joint QoS routing and channel assignment* problem. Although a few solutions have been proposed for both QoS routing and channel assignment problems in multi-channel multi-radio WMNs, the joint problem has not yet been studied.

The existing algorithms for QoS routing problem [2–12] either do not consider the multi-channel nature of the network or assume that channel assignment is performed before loading the network, and it is fixed. The solutions obtained by these algorithms are suboptimal as they are not capable of adapting network resources according to traffic demands. Furthermore, their performance depends on the channel assignment algorithm.

The proposed channel assignment schemes in the literature are classified into two broad categories: static and dynamic[1] [13, 14]. In the former category, channels are assigned for a long period of time while in the latter, channels may be reassigned frequently over time according to needs. Static methods are oblivious to dynamics of network traffic; consequently, they give suboptimal network performance. On the other hand, dynamic approaches aim to achieve better performance by adapting network resources for traffic demands. However, existing dynamic channel assignment algorithms [15–20] do not consider end-to-end QoS requirements of flows and are not coupled with routing.

In this paper, we study the *on-line* joint QoS routing and channel assignment problem. In this problem, it is assumed that each demand arrives at a particular time and requires a specific bandwidth. The demand is accepted if we can find a path with sufficient bandwidth, otherwise it is rejected. The primary goal is to maximize acceptance rate of the demands by jointly optimizing routing and channel assignment. We assume that routing and channel assignment are parts of the network management tool, so they are centralized algorithms and run on the call admission control (CAC) server, which has a fairly accurate

[1]Fast switching is a special case of the dynamic approaches in which channels are changed per-packet. The method needs particular MAC protocol and is not considered in this paper.

and complete view of the network. It should be noted that in spite of existing many solutions for the joint routing and channel assignment problem, they are not applicable to this problem because they do not consider end-to-end QoS requirements and are off-line schemes.

Our contributions to the on-line joint QoS routing and channel assignment problem are as follows.

- We formulate the problem and identify the design requirements of the algorithms for QoS routing and channel assignment subproblems.
- We design the QoS Driven Dynamic Channel Assignment (QDDCA) algorithm as an efficient resource management tool to adapt network resources according to traffic demands.
- We develop a k-shortest path based on-line QoS routing algorithm. This algorithm and QDDCA are integrated in the Joint QoS Routing and Channel Assignment (JQRCA) algorithm to provide an efficient solution for the problem.
- We propose a technique to obtain an upper bound on the network performance. We develop an optimal mixed integer linear programming (MILP) model for an artificial version of the problem, in which demands are static. Due to intractability of the model, we relax it to get an upper bound. By extending the relaxed model to dynamic demand case, we model the original problem. Since it leads to an enormous model; we develop a decomposition algorithm which splits the problem into many small subproblems and solves them sequentially.

The remaining of this paper is organized as follows. In Section 2, we review the related work and highlight shortcomings of existing solutions to apply them on this problem. Assumptions, system models, and problem statement are presented in Section 3. We explain the main ideas of our solution in Section 4. The QDDCA algorithm is presented in details in Section 5. Section 6 explains the JQRCA algorithm. The technique to obtain an upper bound on the network performance is explicated in Section 7; moreover, in this section, we present the simulation results to show the efficiency of the technique. Simulation results to evaluate the performance of JQRCA under various settings of network and traffic parameters are presented in Section 8. Finally, Section 9 concludes this paper.

2. Related Work

In this section, we review three categories of related work including QoS routing algorithms in WMN, dynamic channel assignment schemes, and solutions proposed for the joint routing and channel assignment problem.

There are a number of studies on the problem of finding feasible path in WMN [2–5] since it is *NP-Complete* in multi-hop wireless networks [21, 22]. A genetic algorithm was proposed in [2] and in [3–5], flooding based algorithms were developed. The key issues in this problem are to estimate link available bandwidth and control admission of demands, which have been studied in [6–8]. However, these solutions only focus on finding a feasible path and do not consider the network performance optimization problem.

The problem of optimizing network performance has been studied in [9–11]. In [9], the authors proposed a routing metric to find the cost-effective paths. The proposed routing metric in [10] considers link available bandwidths and channel diversity. A hop-count bounded heuristic algorithm was proposed in [11] that finds the feasible path with the maximum bottleneck capacity. Although these solutions attempt to maximize network performance, they assume that channel assignment is fixed; thus, their performance depends on the given channel assignment. The authors in [11] and [12] considered the channel assignment problem besides QoS routing, but they did not solve the joint problem. In both solutions, there are two phases; in the first phase, a static load-unaware channel assignment is performed and the second phase is QoS routing.

The previous work on dynamic channel assignment in multi-channel multi-radio WMN can be viewed in two categories [13]: the approaches designed to mitigate external interference [15–17] and the solutions that reassign channels based on local load measurements [18–20]. In the first category, there is an external source of interference, nodes measure interference periodically, and switch to the least interfered channel. Although minimizing the external interference improves network performance, this category does not explicitly consider network traffic, its dynamics, and QoS requirements. In the second category, each node measures its link loads and if detects an overloaded link, changes the channel of the link. These solutions attempt to improve the *one-hop* capacity of the network but cannot guarantee the end-to-end bandwidth requirement of flows, which is the main constraint in supporting QoS.

Combinations of channel assignment and other problems, including routing, scheduling, and power control have been the subject of many studies [23–34]. The goal of these joint problems is to maximize network throughput subject to a fairness constraint. The number of adjustable parameters is the factor makes the difference between these studies. A group combined routing and channel assignment [23–27], while some others studied the joint problem of routing, channel assignment, and scheduling [28–31]. Another group even took the power and/or rate control into account [33, 34].

We have a closer look at the joint routing and channel assignment algorithms [23–27]; the second and third groups are beyond the scope of this paper. In [23], an iterative algorithm was proposed; for a given set of flows, the algorithm iteratively adjusts routing and channel assignment as long as it can improve network throughput. The authors in [24] developed a simulated annealing based method to find the optimal channel assignment and routing. The idea of the solution in [25] is to split a large optimization problem into many small subproblems. The subproblems are solved independently, and the final feasible solution is obtained after post processing. The architecture proposed in [26] uses multipath routing and meanwhile attempts to minimize the interference between multiple paths of each flow. The joint routing and channel assignment problem was modeled as a non-linear mixed integer problem in [27]; after linearization, the authors used the dual decomposition methods to find a near optimal solution.

These solutions are not applicable to the on-line joint QoS routing and channel assignment problem for the following reasons. First, the desired objective, maximizing per-flow achievable rate, is different from the goal of the joint QoS routing and channel assignment in which the number of admitted demands should be maximized. Second, these solutions are *off-line*; they need information of all flows at the beginning. Third, when traffic pattern changes, e.g., a

Table 1: Notations

Notation	Description
V	Set of nodes and $\|V\| = n$.
E	Set of edges and $\|E\| = m$.
Δ	Set of demands, $\Delta = \{\delta_i = (s_i, d_i, b_i, t_i, e_i)\}$, and $\|\Delta\| = h$.
K	Set of channel, $\|K\| = \kappa$.
T_R	Transmission range
I_R	Interference range, $I_R = T_R \times q$ and $q > 1$.
r_u	The number of radios of node u
p	A path in the network
Ψ	Channel assignment pattern
(u, v)	Link $(u, v) \in E$
$c^k_{(u,v)}$	Physical channel capacity of (u, v) on channel k
$I_{(u,v)}$	Interference set of link (u, v)
$I'_{(u,v)}$	$I_{(u,v)}$ when the same channel is assigned to all links
$\hat{I}$	Size of the largest interference set
$w^{\Psi}_{(u,v)}$	Weight of link (u, v) under channel assignment Ψ
$l_{(u,v)}$	Total load on link (u, v)
$l^k_{(u,v)}$	Load on link (u, v) on channel k
$f^i_{(u,v)}$	Flow of δ_i on link (u, v)
Φ	The set of existing flows

new flow is added, these algorithms may change all already assigned channels and reroute all flows that lead to a significant overhead to update entire network.

3. System Model and Problem Statement

In this section, first, we describe the assumptions and system models; then, the problem considered in this paper is formulated. Notations used through the paper are denoted in Table 1.

3.1. Assumptions

We consider IEEE 802.11 based multi-channel multi-radio wireless mesh networks. In the network, all nodes are static, have multiple radios and all radios have the same transmission range T_R and interference range I_R. It is supposed that the RTS/CTS mechanism is enabled. It is assumed that there are κ *orthogonal* channels and the adjacent channel interference is negligible due to proper design and implementation of wireless network interface cards and sufficient spectral separation between the channels [11, 15, 16, 18, 20, 23, 28, 29, 32]. The physical channel capacity of link (u, v) on channel k is $c^k_{(u,v)}$ Mb/s. Detailed measurements in WMNs reported in [35] showed that the PHY layer is stable and predictable; hence, we use the abstract model and assume that the physical channel capacity does not vary over time. We assume that each link can transmit on only one channel at any given time, flows are not splittable, and radios have not fast switching capability.

3.2. Network Model

Network is modeled by a digraph $G = (V, E)$, where V is a set of n vertices and E is a set of edges. Each $v \in V$ corresponds to a node in the network. Suppose $d(u, v)$ is the Euclidean distance between u and v. For a given pair of nodes u and v, there is a link $(u, v) \in E$ if and only if $d(u, v) \leq T_R$.

3.3. Interference Model

We use the *interference range* model [36], which is a special case of the protocol model [37]. This model, in conjunction with the RTS/CTS mechanism, yields that links (u_1, v_1) and (u_2, v_2) interfere with each other if the same channel is assigned to both of them and if the sender or receiver of one of them is in the interference range of the sender or receiver of the other one [11, 16, 28]; more specifically, $d(u_1, u_2) \leq I_R$ or $d(u_1, v_2) \leq I_R$ or $d(v_1, u_2) \leq I_R$ or $d(v_1, v_2) \leq I_R$. $I_{(u,v)}$ is the set of the links that interfere with (u, v). By definition (i) $(u, v) \in I_{(u,v)}$, (ii) $(u_1, v_1) \in I_{(u_2,v_2)}$ if and only if $(u_2, v_2) \in I_{(u_1,v_1)}$, and (iii) $I_{(u,v)}$ corresponds to neighbors of (u, v) in the link interference/contention graph. We denote the interference set of (u, v) by $I'_{(u,v)}$ when the same channel is assigned to all links in the network. Note that $I'_{(u,v)}$ contains all the links in the interference *rage* of (u, v).

3.4. Available Bandwidth Model

The authors in [38] proposed two sufficient conditions for feasibility of bandwidth allocation in multi-hop wireless networks: the *row* constraint and the *scaled clique* constraint. In the following, we explain the row constraint; the scaled clique constraint is discussed in more details in Section 7.1.2.

Let Φ denote the set of exiting flows in the network that specify the load on each link, $l^k_{(u,v)}$. The row constraint enforces that

$$\sum_{(a,b)\in I_{(u,v)}} \frac{l^k_{(a,b)}}{c^k_{(a,b)}} \leq 1 \qquad \forall (u,v) \in E, \tag{1}$$

where k is the channel assigned to (u, v) and (a, b). In (1), $\frac{l^k_{(a,b)}}{c^k_{(a,b)}}$ is the fraction of time (a, b) needs to transmit load $l^k_{(a,b)}$. Hence, the row constraint imposes that the aggregate transmission time in each interference set should be less than or equal to one. Throughout this paper, we refer (1) as the *capacity constraint*. By satisfying the capacity constraint, we ensure that the physical capacity of each link, $c^k_{(u,v)}$, is sufficient to carry the load, $l^k_{(u,v)}$, subject to the interferences. Consequently, the bandwidth allocation for the set Φ of existing flows is feasible, all the flows can be transmitted at the desired rate, and their required bandwidth is guaranteed. Using the capacity constraint (1), the available bandwidth of a link is defined as follows.

Definition 1. *Suppose that the set of existing flows is denoted by Φ; in this case, available bandwidth of (u, v) on channel k is* $ALB^k_\Phi(u, v) = c^k_{(u,v)}\Big(1 - \sum_{(a,b)\in I_{(u,v)}} \frac{l^k_{(a,b)}}{c^k_{(a,b)}}\Big)$.

Note that satisfying (1) implies $1 - \sum_{(a,b)\in I_{(u,v)}} \frac{l^k_{(a,b)}}{c^k_{(a,b)}} \geq 0 \ \forall (u, v) \in E$ that means $ALB^k_\Phi(u, v) \geq 0 \ \forall (u, v) \in E$. Thus, the last inequality is a sufficient condition for feasibility of bandwidth allocation for the set Φ of *existing flows* in the network.

3.5. Problem Statement

The problem studied in this paper is to optimize network performance, which is measured in terms of acceptance rate of demands with QoS constraints. In the problem, there is a set of demands $\Delta = \{\delta_i = (s_i, d_i, b_i, t_i, e_i)\}$ in which, demand δ_i arrives at time t_i, needs a path with bandwidth b_i from node s_i to node d_i. If it is admitted, it will leave the network at time e_i. A *feasible path* from s to d needs to be found to admit demand δ; it is a path that allocating the required bandwidth b through it does not violate the capacity constraint (1) of any link. Let Φ denote the set of existing flows before the arrival of δ and $\Phi' = \Phi \cup \delta$. In the wired network, $ALB^k_\Phi(u, v) > b \; \forall(u, v) \in p$ is the necessary and sufficient condition for feasibility of the path p for demand δ[2]. However, in wireless networks, due to the intra-flow interference, a demand may use the available bandwidth of each link multiple times; moreover, because of the inter-flow interference, a demand uses the bandwidth of the links which are *not* in the path of the demand. Hence, $ALB^k_\Phi(u, v) > b$ is a necessary but not sufficient condition. The sufficient condition for feasibility of a path p for demand δ is $ALB^k_{\Phi'}(u, v) > 0 \; \forall(u, v) \in E$, which means that the capacity constraint (1) is satisfied for all links *after* allocating the bandwidth b for demand δ that creates the new set Φ' of existing flows[3].

Note that the network performance optimization problem is, in fact, the problem of maximizing the probability of existence of feasible paths. Resource availability in the network is the main factor that affects existence of feasible paths. The factor is influenced by routing and channel assignment algorithms, which act as the resource *consumer* and *producer*, respectively. Routing algorithm determines how network resources are consumed by flows and channel assignment algorithm, according to definition 1, specifies the available bandwidth of each link. There is an interaction between these algorithms; routing algorithm selects paths according to the resources that are specified by channel assignment; on the other hand, if routing algorithm needs additional bandwidth on a link, channel assignment algorithm can provide it by rearranging channels. In summary, to maximize the probability of existence feasible paths, routing and channel assignment should be jointly optimized.

In this paper, we consider the *on-line* version of the problem in which there is not any information about a demand before it arrives. At the demand arrival time, CAC decides to accept the demand or not. The admission strategy can be *greedy* or *non-greedy*. In the former strategy, each demand is accepted if and only if there is a feasible path for it. However, in the latter, CAC may decide to reject a demand in spite of existence of a feasible path for some reasons, e.g., because the demand is very resource intensive. Here, we consider the greedy strategy. It is appropriate to maintain (absolute) fairness since it aims to admit every demand disrespect of its bandwidth requirement. Moreover, we assume that it is not allowed to reroute the flows in the networks, whereas we use channel reassignment to adapt network resources dynamically.

[2]For wired network, we have $I_{(u,v)} = \{(u, v)\}$.

[3]Note that in wired networks, $ALB^k_\Phi(u, v) > b \; \forall(u, v) \in p$ implies that $ALB^k_{\Phi'}(u, v) > 0 \; \forall(u, v) \in E$; hence, this is also a sufficient condition in wired networks.

4. Solution Approach and Design Requirements

Our proposed solution for the problem is an iterative algorithm that consists of two phases: finding a path and maintaining its feasibility. The solution is an integration of two algorithms, a routing algorithm to find a path and an on-demand channel (re)assignment algorithm to maintain feasibility of the path. The main idea behinds the solution is that channel assignment can be used as an effective resource management tool to adapt network resources according to the needs of the network traffic. Based on this idea, the core of the iterative algorithm is as follows. For a given demand, the routing algorithm finds a *not necessarily feasible* path. If the path is infeasible, the channel assignment algorithm attempts to rearrange channels to make the path feasible; if it fails, another path is found and so on. This iteration continues until the demand is accepted by finding a feasible path or some other criteria are met. Details of these algorithms will be explained in Sections 5 and 6. In the following of this section, we identify the design requirements of each algorithm; satisfaction of the requirements is discussed in Sections 5.1 and 6.1.

To design the routing and channel assignment algorithms, three sorts of issues should be considered. The first issue is to achieve the performance optimization goal, maximizing acceptance rate of demands. For this purpose, the routing algorithm should select optimal paths, and the channel assignment algorithm needs to adapt network resources according to traffic demands.

The second issue is the interaction between these algorithms. The routing algorithm must be aware of the capabilities of the channel assignment algorithm. Since the path found by the routing algorithm is not necessarily feasible, it should avoid selecting infeasible paths that cannot be made feasible by the channel assignment algorithm. On the other hand, the channel assignment algorithm should take into account the optimality of the path found by the routing algorithm because the routing metric used by the routing algorithm can be a function of (available) bandwidth and/or interference, and these parameters depend on channel assignment. Hence, the channel reassignment strategy must be *consistent* with the routing metric; in other words, channels selected by the channel assignment algorithm must not contradict optimizing path weights, which is aimed by the routing algorithm.

Third, it is preferred to use local information in both routing and channel reassignment; using the whole global network information to compute routing metric or reassign channels leads to high computational complexity which is unacceptable. Besides the information locality, channel reassignment must also maintain *impact locality*, which implies a channel reassignment of link should not propagate in the whole network and should not influence other links far away from the link. Satisfying the information locality does not necessarily guarantee the impact locality because changing channel of a link may trigger many other reassignments in the network due to the channel dependency and limited number of radios, which is known as the ripple effect [18].

Besides these requirements, the number of channel reassignments should be minimized. This is necessary to reduce the overhead of the algorithm and amount of the signaling traffic used to update channels in the network.

5. QoS Driven Dynamic Channel Assignment

As discussed in the previous section, channel assignment is the second phase of our proposed solution. It runs if the path found by the routing algorithm is not feasible. The input of the channel assignment problem is a demand routed through a path p and the objective is to make the path feasible if it is not.

In this section, we first clarify the design choices in the channel (re)assignment algorithm. Then, we explain how they help us to meet the requirements mentioned in Section 4, and finally, we present the QoS Driven Dynamic Channel Assignment (QDDCA) algorithm and its computational complexity analysis.

5.1. Design Choices

There are four design decisions in the channel (re)assignment algorithm: channel reassignment strategy, best channel selection metric, group channel change technique, and resource utilization strategy. In the following, we clarify our choices for these decisions.

5.1.1. On-demand Channel Reassignment

Our channel reassignment strategy is on-demand; channels are changed only if the path found by the routing algorithm is not feasible under current channel assignment. As explained in Section 3.5, $ALB^k_{\Phi'}(u,v) > 0 \ \forall (u,v) \in E$ is the sufficient condition for feasibility of the path, where Φ' is the set of flows, including the new demand. Therefore, infeasibility of the path implies that allocating the required bandwidth through the path violates the capacity constraint (1) of at least one link; in other words, $\exists (u,v) \in E$ s.t. $ALB^k_{\Phi'}(u,v) < 0$. The link for which its capacity constraint is violated is named *violated link*; it is the key concept in our proposed solution.

The main body of the on-demand algorithm is as follows. For a given path, we check feasibility of the path. If the path is feasible, the demand is accepted; otherwise, we find the violated links and change their channels. The new channel for each violated link is the *best feasible* channel. Satisfying feasibility and finding the best channel are explained in the following.

Note that violated links are not necessarily in the path; even, it is possible that none of the links in the path is violated while there are some other violated links in the network. Fig. 1 illustrates this issue. Assume a channel with capacity 100 is assigned to all links. In this figure, interference range of nodes b and g are shown by dashed circles; so, $I_{(a,b)} = I_{(b,c)} = \{(a,b),(b,c),(d,e)\}$ and $I_{(d,e)} = \{(a,b),(b,c),(d,e),(f,g)\}$. Two flows, one from d to e and another from f to g, are already admitted and now, there is a new traffic demand from a to c. If the required bandwidth 20 is allocated on links (a,b) and (b,c), the capacity constraint of links (a,b) and (b,c) are satisfied, $\frac{l_{(a,b)}}{100} + \frac{l_{(b,c)}}{100} + \frac{l_{(d,e)}}{100} < 1$, but the constraint of (d,e) is not, $\frac{l_{(a,b)}}{100} + \frac{l_{(b,c)}}{100} + \frac{l_{(d,e)}}{100} + \frac{l_{(f,g)}}{100} > 1$; thus, the out-of-path link (d,e) is violated, whereas the in-path links (a,b) and (b,c) are not.

5.1.2. Feasibility Satisfaction

A feasible channel assignment needs to satisfy the *capacity* and *radio* constraints. The capacity constraint is defined by (1) and the radio constraint enforces that the number of channels assigned to the links of node u must be at most r_u. Suppose link (u,v) is violated, and we want to assign a new channel

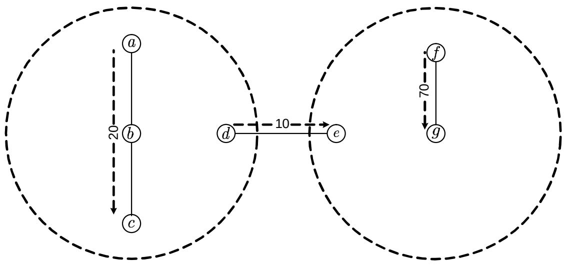


Figure 1: Illustration of out-of-path violated links. Interference ranges and flows are shown by dashed circles and dashed arrows, respectively. The same channel with capacity 100 is assigned to all links. The new flow from a to c violates capacity constraint of out-of-path link (d, e).

to the link. It is easy to see that the radio constraint at node u is satisfied if at least one of the following conditions holds: (i) a radio of u is already tuned to the new channel, or (ii) the old channel assigned to (u, v) can be replaced by the new channel, or (iii) there is a free radio in the node. To avoid the ripple effect [18], the second condition holds only if no link except (u, v) uses the old channel.

Radio consumption to switch to the new channel depends on the satisfied condition. Satisfaction of the first condition not only needs no extra radio, but also it implies that the radio tuned to the old channel can be freed if no other link uses the channel. In case of satisfaction of the second condition, once again, no extra radio is needed but no radio can be freed because the radio tuned to the old channel now is used by the new channel. If the third condition is true, not only no radio can be freed but also an extra radio is used for the new channel. Therefore, to minimize radio consumption, these conditions are checked in the aforementioned order, and the radio constraint is satisfied as soon as one of the conditions is true.

According to these constraints, we define two types of channels as follows.

Definition 2. *Candidate channel for a link is a channel that satisfies the radio constraint in both nodes of the link.*

Definition 3. *Valid channel is a candidate channel that also satisfies the capacity constraint.*

5.1.3. Best Channel Selection

When there is more than one valid channel for a violated link, the best one should be selected. As we mentioned earlier, it affects the optimality of the path found by routing algorithm and hence must be consistent with routing. Let $w^{\Psi}_{(u,v)}$ be the weight of link (u, v) under channel assignment Ψ. If $w^{\Psi}_{(u,v)}$ depends on interference, $I_{(u,v)}$, or bandwidth, $ALB^{k}_{\Phi}(u, v)$, changing channel assignment from Ψ to $\overline{\Psi}$ modifies link (and consequently, path) weights.

Routing algorithm finds an optimal path under channel assignment Ψ by minimizing the weight of the *path*, $W(p, \Psi)$, which is the sum of the weight of the links in the path, $W(p, \Psi) = \sum_{(u,v)\in p} w^{\Psi}_{(u,v)}$. To be consistent with routing, we define the best channel as the channel that if assigned to the violated link minimizes the weight of the *network* under new channel assignment $\overline{\Psi}$, $W(G, \overline{\Psi})$, which is the sum of the weight of all links, $W(G,\overline{\Psi}) = \sum_{(u,v)\in E} w^{\overline{\Psi}}_{(u,v)}$. Due to this definition, the computational complexity of finding the best channel is proportional to $O(m)$. However, if routing metric is based on local information, minimizing $W(G,\overline{\Psi})$ is accomplished with considerably lower computational complexity. In the special case, if we enforce the routing metric to use only the information of the links in the interference set of each link, $w^{\Psi}_{(u,v)} \propto I_{(u,v)}$, we can find the best channel with computational complexity $O(\hat{I})$, where $\hat{I}$ is the size of the largest interference set. In this special case, changing channel of a link at most affects the weight of the links in its interference range. It is easy to show that if new channel assignment $\overline{\Psi}$ is obtained from channel assignment Ψ by changing the channel of link (u,v), we have

$$\begin{aligned} \min W(G,\overline{\Psi}) &= \min \Big(W(G,\Psi) + \sum_{(a,b)\in I_{(u,v)}\cup \overline{I}_{(u,v)}} w^{\overline{\Psi}}_{(a,b)} - \sum_{(a,b)\in I_{(u,v)}\cup \overline{I}_{(u,v)}} w^{\Psi}_{(a,b)} \Big) \\ &= \min \Big(\sum_{(a,b)\in I_{(u,v)}\cup \overline{I}_{(u,v)}} w^{\overline{\Psi}}_{(a,b)} - \sum_{(a,b)\in I_{(u,v)}\cup \overline{I}_{(u,v)}} w^{\Psi}_{(a,b)} \Big), \end{aligned} \quad (2)$$

where $\overline{I}_{(u,v)}$ is the interference set of (u,v) under channel assignment $\overline{\Psi}$. In (2), the second term in the right-hand side of the first line is the aggregate weight of the links in $I_{(u,v)}$ and $\overline{I}_{(u,v)}$ after changing the channel of (u,v) and the third term is the aggregate weight before the channel reassignment. Equation (2) implies that we need to compute the difference between these two aggregate weights, which is a local computation with complexity $O(\hat{I})$. The best channel is the one that gives the minimum value of the difference.

5.1.4. Group Channel Change

The aforementioned procedure to resolve violations focuses on the violated links and attempts to find the best valid channel for the links. However, there are situations, in which although there is not any valid channel for a violated link, changing the channel of the links in its interference set resolves the violation. An example is shown in Fig. 2. Assume that there are two available channels in the frequency spectrum and the physical channel capacities are 100. In this figure, interference ranges of nodes c, d, and f are shown by dashed circles. There are four already admitted flows in the network: (i) form a to b, (ii) from e to f, (iii) from g to h, and (iv) from k to l. In this example, allocating the required bandwidth 30 for the new traffic demand from c to d violates capacity constraint of link (c,d), $\frac{l_{(c,d)}}{100} + \frac{l_{(e,f)}}{100} + \frac{l_{(g,h)}}{100} + \frac{l_{(k,l)}}{100} > 1$. There is not any valid channel for the violated link because both channels are already overloaded in the interference range of (c,d). However, if we assign channel 1 to links (e,f) and (g,h), the violation of (c,d) is resolved. This strategy of channel reassignment is called *Group Channel Change*.

This strategy has a side effect; channel reassignments to resolve a violated link may affect the available bandwidth of other links beyond the interference range of the violated link; e.g., in Fig. 2, resolving the violation of (c,d) affects $ALB^1_{\Phi}(i,j)$ where $(i,j) \notin I_{(c,d)}$. To control the side effect and maintain the impact locality, we propose a group channel change procedure that limits channel reassignments in range $2I_R$ of path p; the procedure is allowed to

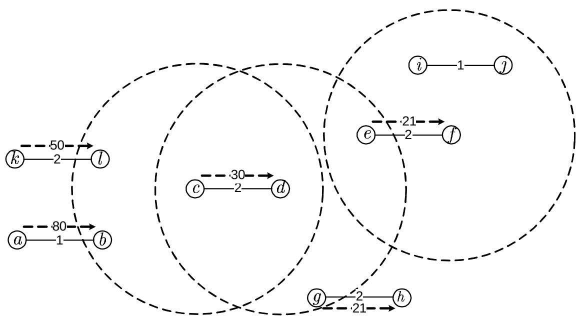


Figure 2: Illustration of group channel change. Interference ranges and flows are shown by dashed circles and dashed arrows, respectively. Links and assigned channels are shown by solid lines. (c, d) is a violated link and changing channel of (e, f) and (g, h) to channel 1 resolves the violation.

change the channel of link (u_3, v_3) if $\exists(u_2, v_2), (u_1, v_1)$ s.t. $(u_3, v_3) \in I_{(u_2,v_2)}$, $(u_2, v_2) \in I_{(u_1,v_1)}$, and $(u_1, v_1) \in p$.

Our *recursive* procedure is as follows. We distinguish between the in-path violated links and the out-of-path ones. If violated link (u_2, v_2) is out-of-path, we change channels of links $(u_3, v_3) \in I_{(u_2,v_2)}$ one-by-one that reduces the number of interfering links with (u_2, v_2) and, as a result, increases $ALB^k_\Phi(u_2, v_2)$. When violated link (u_1, v_1) is in-path, we can move violation from the link to other links $(u_2, v_2) \in I_{(u_1,v_1)}$. For each *candidate channel* of (u_1, v_1), we assign the channel to the link, since it is not a *valid channel*, this assignment violates capacity constraints of some links $(u_2, v_2) \in I_{(u_1,v_1)}$. Now, we have a new set of violated links and attempt to resolve these violations. Note that this procedure creates a loop because if there is not any valid channel for a new violated link, the group channel change procedure is reapplied on the link and if the link is in-path, the procedure creates another new set of violated links and so on. Hence, we do not move violation of the new violated links to other links to avoid the loop; in other words, we treat them as out-of-path links.

5.1.5. *On-demand Resource Utilization and Initial Channel Assignment*

Available channels in frequency spectrum and radios are scarce resources in multi-channel multi-radio WMNs. We assign a channel to a link only if it is in the path of a flow to utilize the resources efficiently. When a flow leaves the network, we check all the links in its route. If there is not any flow routed through link (u, v) on channel k, we remove the channel from the link and check radios of nodes u and v; at each node, if no link uses channel k, we free the radio tuned to the channel.

The main advantage of this strategy is that it increases the probability of existence of free radios, which directly improves the probability of finding feasible paths. Suppose (u, v) is a violated link and both nodes u and v have a free radio; in this case, the set of candidate channels for the link contains all available channels that boosts the probability of existence of at least one valid channel.

To remove the channel of a link, we assign virtual channel 0 to it, which has the following features. First, its physical capacity is 0; routing any flow along a link on channel 0 makes the link violated. Second, interference set of a link

Table 2: Relation between design choices and requirements of channel assignment algorithm

Goal	Choices			
	On-demand Reassignment	Best Channel Selection	On-demand Utilization	Group Channel Change
Maximizing acceptance rate	✓		✓	✓
Minimizing # of channel reassignments	✓			
Routing consistency		✓		
Locality		✓		✓

on channel 0 contains only the link. Third, assigning the channel to a link does not consume any radio. In the initial channel assignment, when there is not any load, all links are assigned to channel 0.

In real applications, to maintain network connectivity, which is required for signaling traffic even when there is not any load to/from a node, the virtual channel 0 can be a default channel. To remove the channel of a link, we *temporarily* assign the default channel to it. If it is impossible due to the radio constraint, it implies that some channels have been assigned to the links of the node; thus, the node is already connected to the network.

5.2. *Achieving Design Goals*

These proposed design choices help us to satisfy the design requirements mentioned in Section 4. Table 2 shows the relation between the design choices and requirements. Acceptance rate is boosted by on-demand channel reassignment that resolve violations, on-demand resource utilization, which frees channels and radios, and group channel change that offers more opportunities to resolve violations. The number of channel reassignments is kept small by the on-demand channel reassignment strategy as it reassigns channels only if it is needed. The routing consistency requirement is met by the best channel selection technique that selects channels according to the routing metric. The proposed solution is localized since selecting the best channel needs local information as long as the routing metric is localized and the group channel change mechanism limits channel reassignments in range $2I_R$ of routing path.

5.3. *QDDCA Algorithm*

The aforementioned design choices are integrated in the QoS Driven Dynamic Channel Assignment (QDDCA) algorithm. Pseudo-code of the algorithm is shown in algorithms 1–4. For a given demand (s, d, b, t, e) routed through path p, QDDCA checks feasibility of bandwidth allocation. If the path is not feasible, it finds violated links and calls ResolveViolation. For each violated link, ResolveViolation first tries to resolve the violation using LinkChannelChange, which assigns the best valid channel to the link if it exists; if LinkChannelChange cannot resolve the violation, GroupChannelChange is invoked in line 5. Since changing channel of a link can resolve multiple violations, after each successful resolve, the remaining violated links are rechecked in line 9.

In group channel change, as mentioned before, we distinguish between in-path and out-of-path violated links. GroupChannelChange in line 1 checks that if the link is out-of-path or is created by the GroupChannelChange itself. If at least one of these conditions holds, it changes the channel of the links in the interference set of the violated link in line 3. If both conditions in line 1 are false, GroupChannelChange checks each candidate channel for the violated link, in lines 7–9, by assigning it to the link, finding new violated links, and attempting to resolve the new violations.

Algorithm 1 : QDDCA($(s, d, b, t, e), p$)

```
1: Check allocating bandwidth $b$ through path $p$
2: if path $p$ is feasible then
3:     return Accept
4: else
5:     $VL \leftarrow$ Violated Links
6:     ResolveViolation($VL$)
7:     if violations were resolved then
8:         return Accept
9:     else
10:        return Reject
```

Algorithm 2 : ResolveViolation(VL)

```
1: while $VL$ is not empty do
2:     $(u, v) \leftarrow VL[0]$
3:     LinkChannelChange($u, v$)
4:     if violation was not resolved then
5:         GroupChannelChange($u, v$)
6:     if violation was not resolved then
7:         return Reject
8:     else
9:         Remove unviolated links from $VL$
```

Algorithm 3 : LinkChannelChange(u, v)

```
1: $VC \leftarrow$ Valid Channels for $(u, v)$
2: if $VC$ is not empty then
3:     Update the channel of $(u, v)$ to the best channel
```

5.4. Worst Case Computational Complexity

The worst case running time of the QDDCA algorithm is the case that all links in path p are violated and LinkChannelChange cannot resolve the violations. In this case, for each link, we have to call GroupChannelChange, wherein lines 5–9 run and ResolveViolation is recalled for the newly generated violated links. In the worst case, LinkChannelChange again cannot resolve the new violations and we have to call GroupChannelChange again. However, in this case, lines 2–3 run that break the recursive function calls.

To analyze the worst case, we use the notations in Table 3. Let κ be the number of channels, and $\hat{r}$ be the maximum number of radios per node. $O(LCC) = O(\kappa(\hat{r} + \hat{I}))$ as we need to check the radio and capacity constraints per channel. $O(GCC_1) = O(LCC)\hat{I} = O(\kappa\hat{I}(\hat{r} + \hat{I}))$. $O(GCC_2) =$

Algorithm 4 : GroupChannelChange(u, v)

```
1: if $(u, v)$ is out-of-path or $(u, v) \in NVL$ then
2:     while $(u, v)$ is violated and there is unvisited $(a, b) \in I_{(u,v)}$ do
3:         LinkChannelChange$(a, b)$
4: else
5:     $CC \leftarrow$ Candidate channels for $(u, v)$
6:     for $\forall ch \in CC$ and if $(u, v)$ is violated do
7:         Change channel of $(u, v)$ to $ch$
8:         $NVL \leftarrow$ New Violated Links
9:         ResolveViolation$(NVL)$
```

Table 3: Notation used for computational complexity analysis of QDDCA

Notation	Complexity of Algorithm
$O(QDDCA)$	QDDCA
$O(RV)$	ResolveViolation
$O(LCC)$	LinkChannelChange
$O(GCC_1)$	Lines 2–3 of GroupChannelChange
$O(GCC_2)$	Lines 5–9 of GroupChannelChange

$O(\kappa\hat{r} + \kappa(\hat{I} + \hat{I}(O(LCC) + O(GCC_1)))) = O(\kappa^2\hat{I}^2(\hat{r} + \hat{I}))$ since the radio constraint must be checked for κ channels and at most there would be $\hat{I}$ new violated links that ResolveViolation is called for. The length of path can be at most n, so $O(RV) = n(O(LCC) + O(GCC_2)) = O(n\kappa^2\hat{I}^2(\hat{r} + \hat{I}))$ and finally $O(QDDCA) = O(n\hat{I}) + O(RV) = O(n\kappa^2\hat{I}^2(\hat{r} + \hat{I}))$.

It should be noted that the worst case occurs very rarely in practice. Our simulations, which are presented in Section 8.7, show that the length of paths is much less than the number of nodes, n, the number of violated links is less than the length of the path, and the number of additional new violated links generated by GroupChannelChange is less than one per violated link.

6. Joint QoS Routing and Channel Assignment

We explained in Section 4 that the first phase of our proposed solution is routing. The input of the routing algorithm is a demand, and the objective is to find a path, which is not necessarily feasible. In this section, we first clarify the design choices, then, discuss how the choices aid us to accomplish the desired design objectives, and finally, we present the Joint QoS Routing and Channel Assignment (JQRCA) algorithm and its computational complexity analysis.

6.1. Design Choices

The major design decisions in the routing algorithm are pruning, search algorithm, and routing metric, which are explained in details in the following.

6.1.1. Pruning

Network pruning is a well-known mechanism in QoS routing to exclude from the search space the links that have not sufficient resources. Contrary to the QoS routing problem, in the joint QoS routing and channel assignment problem, if the current available bandwidth of a link is not sufficient to route a flow through it, the link should not be pruned because it is possible to provide adequate bandwidth for the link through an appropriate channel reassignment.

However, the channel assignment algorithm cannot provide any arbitrary bandwidth; it must obey the physical channel capacity and radio constraints. Since we assume each link can only use one channel, the maximum possible load on link (u,v) can be at most $c^k_{(u,v)}$; this is the best case in which no other link interferes with it. For a given demand $\delta = (s,d,b,t,e)$, link (u,v) is pruned if $l_{(u,v)} + b > \max_{k \in K'} \left\{c^k_{(u,v)}\right\}$, where K' is the set of *candidate channels* for (u,v), because routing the demand through the link makes it violated and the violation cannot be resolved. This inequality also considers the radio constraint; if there is not any candidate channel for a link due to the constraint, $\max_{k \in K'} \left\{c^k_{(u,v)}\right\} = 0$, the link is pruned because routing any demand through the link makes it violated while there is not any possibility to resolve it.

6.1.2. Search Algorithm

To search the network graph, we use the k-shortest path algorithm. There are two reasons for this choice. First, in the previous section, we developed a channel reassignment algorithm that takes a path as the input and reassigns channels to make it feasible. However, it cannot guarantee feasibility of any given path; therefore, instead of examining only one path, we investigate k paths one-by-one to increase the probability of finding feasible paths. Second, the algorithm is adjustable; the number of paths can be used to adjust the trade-off between the running time and the probability of finding feasible path.

We use the k-shortest path algorithm to find only one feasible path; the JQRCA algorithm is a single-path algorithm. Although splitting a flow among multiple paths may increase the probability of finding feasible (multi)path, it has its own complexities. It complicates the design of the algorithm and in real applications causes out-of-order packet reception, which is not acceptable in most cases. Moreover, our simulations in Section 8.3 show that as long as the required bandwidth of flows is not comparable to physical channel capacities, flow splitting and multipath routing are not notably beneficial.

6.1.3. Routing Metric

As we discussed in Section 4, the network performance depends on bandwidth availability in the network. To optimize it, we should minimize bandwidth consumption at each link, which is directly proportional to the size of the interference sets. Thus, we need to find the path with minimum interference that implies the routing metric should be the size of the interference set, $w^{\Psi}_{(u,v)} = |I_{(u,v)}|$. If the channel of a link is the virtual channel 0, we find the size of the interference set for each *candidate channel* of the link and use the average of them as its weight. Note that this routing metric satisfies the locality constraint mentioned in subsection 5.1.3.

6.2. Achieving Design Goals

The proposed choices in the previous section meet the design requirements we identified in Section 4. Table 4 shows the relation between the choices and objectives. Network pruning, k-shortest path routing, and the interference based routing metric improve acceptance rate; since, the pruning mechanism excludes the links that cannot be resolved, the k-shortest path algorithm provides more

Table 4: Relation between design choices and objectives of QoS routing

	Choices		
Goal	Pruning	k-Shortest path	Routing metric
Maximizing acceptance rate	√	√	√
Channel assignment awareness	√		
Locality	√		√

opportunity to search the network, and the interference based routing metric aims to minimize resource consumption by each demand. The channel assignment awareness requirement is met by the pruning algorithm as it considers the capabilities of the channel assignment algorithm and excludes the links that the algorithm cannot resolve. Since both the pruning mechanism and the routing metric use the local information of each link, the proposed solution is localized.

6.3. JQRCA Algorithm

As we mentioned, our solution iteratively finds a path and attempts to make the path feasible. It is implemented by integrating the k-shortest path algorithm and QDDCA. Pseudo-code of the algorithm is shown in algorithm 5.

To find k paths, k instances of each node except the source node are initialized and added to the list L in lines 1–2. $u[i].w$ and $u[i].\pi$ are the weight and parent of instance $u[i]$, respectively. In the main loop of the algorithm, the minimum weight instance $u[i]$ is selected by GetBestInstance and the partial path p' from the source node to node u is found by GetPartialPath. If node u is the destination, we have found a path; therefore, in lines 7–9, we check feasibility of the path, reassign channels if it is required, and finish the algorithm by accepting the demand in the case of feasibility of the path. If u is not the destination, we need to update the weight of the instances of the neighbors of u. An instance $v[j]$ is updated if (u, v) is not pruned, v is not in partial path p', and the current weight of the instance, $v[j].w$, is greater than the total weight of link (u, v) and partial path p'.

6.4. Worst Case Computational Complexity

We assume list L is implemented by the Fibonacci heap, so $O(\textsc{GetBestInstance}) = O(\log(kn))$ and the complexity of initializing the heap in lines 1–2 is $O(kn\log(kn))$. Each part of the main loop of the algorithm runs different times. Lines 4 and 5 run kn times, so total complexity of this part is $O(kn(\log(kn) + n))$. Lines 7–9 run at most k times; the total complexity is $O(O(QDDCA)k) = O(kn\kappa^2\hat{I}^2(\hat{r} + \hat{I}))$. The last part, lines 11–15, runs km times, consequently the total complexity is $O(knm)$. Combining all these running times yields to $O(JQRCA) = O(kn\log(kn)) + O(kn(\log(kn) + n)) + O(kn\kappa^2\hat{I}^2(\hat{r} + \hat{I})) + O(knm) = O(kn\log(kn) + knm + kn\kappa^2\hat{I}^2(\hat{r} + \hat{I}))$.

7. Performance Bound

In this section, we obtain an upper bound on the network performance, which is used in Section 8 to evaluate the performance of the JQRCA algorithm. For the sake of simplicity of presentation, in the first step, we start from an artificial version of the problem in which the QoS demands are *static* and obtain the

Algorithm 5 : JQRCA($(s, d, b, e, t), k$)

```
 1: for $i = 1$ to $k$ do
 2:     $\forall u \in V \setminus \{s\}$, $u[i].w \leftarrow \infty$ and add $u[i]$ to $L$
 3: while the number of found paths to $t$ is less than $k$ do
 4:     $u[i] \leftarrow$ GetBestInstance($L$)
 5:     $p' \leftarrow$ GetPartialPath($u[i]$)
 6:     if $u = t$ then
 7:         QDDCA($(s, d, b, t, e), p'$)
 8:         if Accepted then
 9:             Finish
10:     else
11:         for each $(u, v) \in E$ do
12:             if $(u, v)$ is not pruned and $v \notin p'$ and $\exists v[j]$ s.t. $v[j].w > W(p', \Psi) + w^{\Psi}_{(u,v)}$ then
13:                 $v[j].w \leftarrow W(p', \Psi) + w^{\Psi}_{(u,v)}$
14:                 $v[j].\pi \leftarrow u[i]$
15:                 update $L$
```

optimal feasible solution for it through formulating the problem as a MILP model, OptimalStatic. Due to the computational complexity of the model, we relax it to get an upper bound, RelaxedStatic model. In the second step, we assume that flows are reroutable and extend the relaxed model to consider the dynamics of the demands over the time, DynamicUB1 model. However, it leads to a huge model that is intractable in practical problems. We deal with it by proposing another model, DynamicUB2, which is equivalent to the first model, but it is *decomposable*, and developing a decomposition algorithm, MostGreedyOnline, for it. We show that the models for dynamic demands simulate the behavior of the on-line greedy CAC strategies, which we study here. The solution of the extended model, which is acquired by the decomposition algorithm, is the performance bound of the on-line joint QoS routing and channel assignment problem. Fig. 3 summarize our approach to obtain the upper bound.

7.1. Static Demands Performance Bound

The static demands performance bound problem is as follows. A multi-channel multi-radio WMN, which is modeled by a digraph, and a set of *static* QoS demands are given. By the static demands, we mean all the demands arrive at time 0. The question is *what the maximum number of admissible demands is.* For this problem, we develop an optimal MILP model and since it is extremely difficult, we relax it and obtain a relaxed model that is tremendously easier and provides a tight bound for the problem.

7.1.1. Optimal Model

In the optimal MILP model, we use the assumptions we made in the previous sections; each link can only use one channel, there is not fast switching capability, and the capacity constraint is modeled by the row constraint (1). However, we assume that flows are splittable and multipath routing is used. In addition to the notations in Table 1, we use the following variables. Binary variable $x^k_{(u,v)}$ is the channel assignment variable,

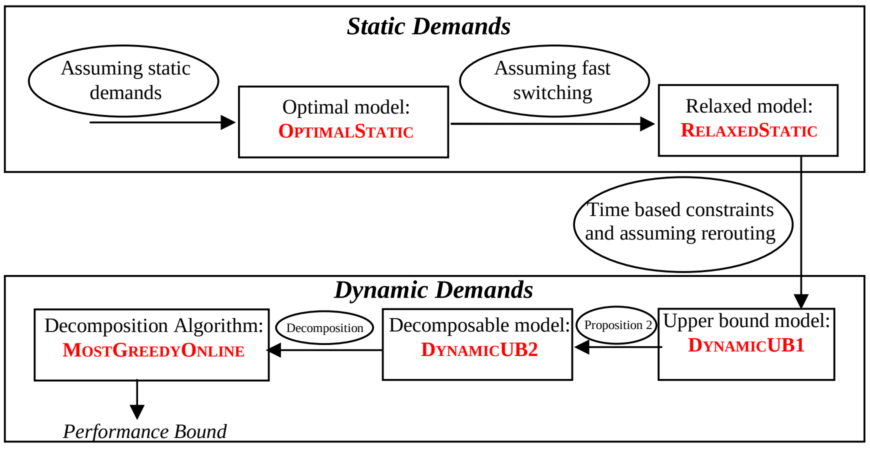

Static Demands
Assuming static demands
Optimal model: OptimalStatic
Assuming fast switching
Relaxed model: RelaxedStatic
Time based constraints and assuming rerouting
Dynamic Demands
Decomposition Algorithm: MostGreedyOnline
Decomposition
Decomposable model: DynamicUB2
Proposition 2
Upper bound model: DynamicUB1
Performance Bound

Figure 3: The proposed approach to obtain network performance bound

$$x^k_{(u,v)} = \begin{cases} 1, & \text{if link } (u,v) \text{ transmits on channel } k \\ 0, & \text{otherwise.} \end{cases}$$

Binary variable a_i denotes admission of demand δ_i,

$$a_i = \begin{cases} 1, & \text{if demand } \delta_i \text{ is accepted} \\ 0, & \text{otherwise.} \end{cases}$$

Tuning radios to channels is modeled by variable y^k_u,

$$y^k_u = 1, \text{ if channel } k \text{ is assigned to a radio of node } u.$$

The optimal model is as follows. Its objective function is to maximize the number of admitted demands,

$$\text{maximize} \sum_{\delta_i \in \Delta} a_i. \tag{3}$$

Since at most one channel is assigned to each link, we have

$$\sum_{k \in K} x^k_{(u,v)} \leq 1 \qquad \forall (u,v) \in E. \tag{4}$$

Obviously, the variable y^k_u cannot be greater than 1, so

$$y^k_u \leq 1 \qquad \forall k \in K,\ \forall u \in V. \tag{5}$$

If link (u,v) uses channel k, the channel must be assigned to a radio in both nodes u and v, therefore

$$x^k_{(u,v)} \leq y^k_u,\ x^k_{(u,v)} \leq y^k_v \qquad \forall k \in K,\ \forall (u,v) \in E. \tag{6}$$

The radio constraint forces that the number of channels assigned to the links of a node must be at most the number of radios of the node; in other words,

$$\sum_{k \in K} y^k_v \leq r_v \qquad \forall v \in V. \tag{7}$$

If link (u, v) transmits a load on channel k, the channel must be assigned to the link. So, we have

$$l^k_{(u,v)} \leq x^k_{(u,v)} c^k_{(u,v)} \qquad \forall k \in K,\ \forall (u, v) \in E. \tag{8}$$

The load transmitted by each link must be equal to the load offered on it by flows in the network,

$$\sum_{k \in K} l^k_{(u,v)} = \sum_{\delta_i \in \Delta} f^i_{(u,v)} \qquad \forall (u, v) \in E. \tag{9}$$

Modeling the capacity constraint is a little complicated. To check the capacity constraint (1), $I_{(u,v)}$ and the channel assigned to (u, v) must be given, but indeed these are determined after solving the optimization model. To deal with this issue, in the optimization model, we use $I'_{(u,v)}$ instead of $I_{(u,v)}$ and check the constraint for all channels, there are κ constraints per link. Recall that $I'_{(u,v)}$ is the interference set of (u, v) when a common channel is assigned to all links in the network. However, only one of the κ constraints should be satisfied and the remaining must be don't-care because if channel k is *not* assigned to (u, v), it is meaningless to impose a limitation on the aggregate load transmitted on this channel in the interference range of the link. This is modeled using the *big* M technique and the constraint is

$$\sum_{(a,b) \in I'_{(u,v)}} \frac{l^k_{(a,b)}}{c^k_{(a,b)}} \leq \big(1 - x^k_{(u,v)}\big) M + 1 \qquad \forall k \in K,\ \forall (u, v) \in E. \tag{10}$$

In (10), if channel k is assigned to link (u, v), $x^k_{(u,v)} = 1$, the right-hand side will be "1", and the constraint imposes that the aggregate load transmitted by the links in the interference *rage* of (u, v), $I'_{(u,v)}$, on channel k must not exceed physical channel capacities. However, for other channels $k' \neq k$ where $x^{k'}_{(u,v)} = 0$, this constraint becomes don't-care since M is a big value. The big value implies that M must be greater than $\sum_{(a,b) \in I'_{(u,v)}} \frac{l^k_{(a,b)}}{c^k_{(a,b)}}$; since $\frac{l^k_{(a,b)}}{c^k_{(a,b)}} \leq 1$ and $|I'_{(u,v)}| \leq \hat{I}$, we need $M > \hat{I}$.

Finally, the routing and flow conservation constraint must be satisfied *if demand is accepted*, which is modeled as

$$\sum_{(u,v) \in E} f^i_{(u,v)} - \sum_{(v,u) \in E} f^i_{(v,u)} = \begin{cases} a_i b_i, & \text{if } u = s_i \\ -a_i b_i, & \text{if } u = d_i \\ 0, & \text{otherwise} \end{cases} \qquad \forall u \in V,\ \forall \delta_i \in \Delta. \tag{11}$$

Note that routing variables $f^i_{(u,v)}$ are real variables because of flow splitting and multipath routing assumptions. The last constraints are the bounds,

$$x^k_{(u,v)} \in \{0, 1\},\ a_i \in \{0, 1\},\ l^k_{(u,v)} \geq 0,\ f^i_{(u,v)} \geq 0,\ y^k_u \geq 0. \tag{12}$$

Putting (3)–(12) altogether provides an optimal model for the static demands performance bound problem that is

Model:	OptimalStatic
Objective:	(3)
Subject to:	(4)–(12).

Table 5: The number of maximal cliques

Node #	Link #	Interference Graph Maximal Clique #
25	126	8
50	234	107
100	656	204

Whereas solving the OptimalStatic model gives an optimal feasible solution, it is extremely difficult. The model cannot be solved easily even for small networks and a few number of demands. The complexity arises from the binary variables $x^k_{(u,v)}$ and a_i. In the following, we deal with the complexity by relaxing this optimal model.

7.1.2. Upper Bound

The binary variable $x^k_{(u,v)}$ used for channel assignment is the source of the difficulty of OptimalStatic. We assume that *radios are capable to do fast switching* to tackle the complexity. Using this assumption, variable $x^k_{(u,v)}$ is relaxed as

$$x^k_{(u,v)} = \text{ Fraction of time that link } (u,v) \text{ transmits on channel } k.$$

However, this relaxation causes a problem. The capacity constraint (10) is a conditional constraint and needs the *binary* variable $x^k_{(u,v)}$. We replace it by the scaled clique constraint to deal with this issue. It enforces that the aggregate load of the links in each *maximal* clique of the interference graph should not exceed the *scaled* physical channel capacity. The clique constraint without scaling is a *necessary* condition[4] and formulated as $\sum_{(u,v)\in Q_i} \frac{l^k_{(u,v)}}{c^k_{(u,v)}} \leq 1$ in multi-rate networks [39], where Q_i is a maximal clique. As shown in [38], to be a sufficient condition, the constraint must be scaled.

There are two issues about the scaling. First, the number of maximal cliques in an arbitrary graph theoretically can be exponential; but, in practice, in the interference graph of multi-hop wireless networks, it is limited and all maximal cliques can be found very easily. Table 5 shows the number of maximal cliques in the interference graph of three random topologies. The maximum time to find all maximal cliques is less than one second in our experiments on an Intel Pentium IV 3.0 GHz machine[5].

Second, the value of the scale should be selected properly. The authors in [38] showed that it depends on the *imperfection ratio* of the interference graph. A recent simulation based study of the imperfection ratios of interference graphs provided two conclusions [41]. First, as the number of nodes increases the value of the scale decreases. Second, scale = 1.0 is a good approximation but to be more conservative, we can use scale $= \frac{1}{1.21} = 0.826$. Based on this study, we use both these values to find two bounds.

Let γ be the scale, Q_i be a maximal clique in the interference graph when a common channel is assigned to all links, and set $\Phi = \{Q_1, Q_2, \ldots\}$ be the set

[4]In fact, it is a sufficient condition only in *perfect* interference graphs.

[5]We used MACE program to enumerate maximal cliques [40].

of the maximal cliques. The relaxed model for the static demands performance bound problem is as follows.

Obviously, variable $x^k_{(u,v)}$ is bounded by 1,

$$x^k_{(u,v)} \leq 1 \qquad \forall k \in K,\ \forall (u,v) \in E. \tag{13}$$

Load transmitted by link (u,v) on channel k is proportional to the fraction of time that the link is active on the channel, so

$$l^k_{(u,v)} = x^k_{(u,v)} c^k_{(u,v)} \qquad \forall k \in K,\ \forall (u,v) \in E. \tag{14}$$

When a link of node u, either (u,v) or (v,u), uses channel k, $x^k_{(u,v)} > 0$, in fact, a radio of the node is tuned to the channel and utilized for that transmission for $x^k_{(u,v)}$ fraction of time. Obviously, total utilization of radios of a node cannot exceed the number of radios; in other words,

$$\sum_{k \in K} \Big(\sum_{(u,v) \in E} x^k_{(u,v)} + \sum_{(v,u) \in E} x^k_{(v,u)} \Big) \leq r_u \qquad \forall u \in V. \tag{15}$$

The scaled clique constraint is

$$\sum_{(u,v) \in Q_i} \frac{l^k_{(u,v)}}{c^k_{(u,v)}} = \sum_{(u,v) \in Q_i} x^k_{(u,v)} \leq \gamma \qquad \forall k \in K,\ \forall Q_i \in \Phi, \tag{16}$$

that imposes the total time allocated to all conflicting links must be less than or equal to γ. The bound constraints are

$$x^k_{(u,v)} \geq 0,\ a_i \in \{0,1\},\ l^k_{(u,v)} \geq 0,\ f^i_{(u,v)} \geq 0. \tag{17}$$

These constraints and objective function (3) gives the relaxed model as

Model:	RelaxedStatic
Objective:	(3)
Subject to:	(9), (11), (13)–(17).

It is important to note that even if the exact value of the scale is used, this relaxed model will be an upper bound because its solution may not be *schedulable*. An example of unschedulable solution is depicted in Fig. 4. In this example, in the first time-slot, nodes a and b activate channel 1 on their radios to transmit the load on link (a,b), the length of this time-slot is half of the scheduling frame, $x^1_{(a,b)} = 0.5$, since the load on the link is 5 and the physical channel capacity is 10. In the second time-slot, channel 2 is activated on the radios of nodes b and c to transmit the load on link (b,c), the length of this time-slot is also half of the scheduling frame, $x^2_{(b,c)} = 0.5$. However, there is not any time-slot to transmit the load on link (c,a) on channel 3 even though all the constraints of the RelaxedStatic model are satisfied. Our simulation results presented in the next subsection show that this issue is not an important matter and RelaxedStatic provides a tight bound.

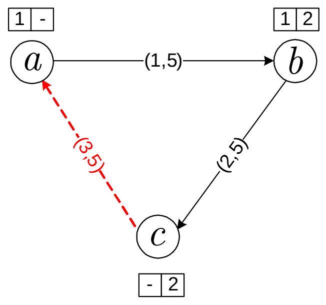


Figure 4: An example of unschedulable solution. Label of each link is (channel, load), label of each node is the schedule of channel activation on its radio, $c^k_{(u,v)} = 10$, and $q_u = 1$. Whereas all the constraints of RELAXEDSTATIC are satisfied, there is not any feasible schedule.

Table 6: Parameters of the topologies used in simulations

Parameter	Values			
Name	T-10	T-15	T-25	T-50
Area	$500{\times}500m^2$	$600{\times}600m^2$	$750{\times}750m^2$	$1000 \times 1000m^2$
Node #	10	15	25	50
T_R	$200m$	$200m$	$200m$	$200m$
I_R	$400m$	$400m$	$400m$	$400m$
Radio #	Random [2,5]	Random [2,5]	Random [2,5]	Random [2,5]
Channel #	12	12	12	12
$c^k_{(u,v)}$	100 Mb/s	100 Mb/s	100 Mb/s	100 Mb/s

7.1.3. Simulation Results

In this subsection, we present simulation results to show the efficiency and tightness of the RELAXEDSTATIC model. We conducted the simulations in three 10, 15, and 25 nodes random topologies with parameters shown in Table 6. In each experiment, 50 random demands were offered to the network. The required bandwidth of each demand was a uniform random variable in $[1, B_{max}]$ Mb/s. We used CPLEX 11.0 [42] on an Intel Pentium IV 3.0GHz machine with 2 Gigabytes RAM. Time limit to solve the model was 10 hours. The results presented in this section are the average of five experiments. We evaluate RELAXEDSTATIC using following metrics.

Definition 4. *Bound Gap of the relaxed model is*

$$\frac{\textit{Relaxed Model Accepted Demands \#} - \textit{Optimal Model Accepted Demands \#}}{\textit{Optimal Model Accepted Demands \#}}.$$

Definition 5. *Time Ratio of the relaxed model is*

$$\frac{\textit{Optimal Model Solution Time}}{\textit{Relaxed Model Solution Time}}.$$

Table 7 shows the simulation results. In this table, rows “Optimal,” “Not Scaled,” and “Scaled” are the results of OPTIMALSTATIC and RELAXEDSTATIC with $\gamma = 1.0$ and $\gamma = 0.826$, respectively. The “Exceed #” row is the number of times that OPTIMALSTATIC was not solved in the specified time limit, in these cases, we used the best integer solution as the result. Optimality gap of the best integer solution, which is reported by the solver, is represented in the “Optimality Gap” row.

Table 7: Simulation results of OptimalStatic and RelaxedStatic. The parameters of the simulation topologies are shown in Table 6.

Topology		T-10		T-15		T-25	
B_{max}		20	30	20	30	20	30
Accepted #	Optimal	48.8	44	49.4	44.4	46.6	40
	Not Scaled	49.3	45.4	49.4	45	49.9	44.8
	Scaled	48.5	42.8	48.8	42.6	48.8	42
Bound Gap	Not Scaled	1.09e−2	3.13e−2	0	1.35e−2	7.09e−2	1.20e−1
	Scaled	−6.15e−3	−2.84e−2	−1.21e−2	−4.05e−2	4.68e−2	5.00e−2
Time(sec)	Optimal	2.17e+4	3.60e+4	1.09e+4	3.60e+4	3.60e+4	3.60e+4
	Not Scaled	9.50e−2	1.73e−1	4.74e−1	6.24e−1	8.71e−1	3.38e+0
	Scaled	1.20e−1	1.12e−1	4.62e−1	8.90e−1	2.16e+0	5.71e+0
Time Ratio	Not Scaled	2.28e+5	2.08e+5	2.30e+4	5.77e+4	4.13e+4	1.07e+4
	Scaled	1.81e+5	3.22e+5	2.36e+4	4.04e+4	1.67e+4	6.31e+3
Exceed #		2	5	1	5	5	5
Optimality Gap		1.66e−2	1.13e−1	6.38e−2	1.27e−1	7.46e−2	2.08e−1

These results lead to the following conclusions. First, RelaxedStatic is a tight relaxation of the optimal model as the bound gap is very small. Second, RelaxedStatic is incredibly, up to 3.22e+5 times, faster than OptimalStatic. Third, the best integer solution is a fairly good approximation of the optimal solution since the optimality gap is quite small. Fourth, these results confirm the conclusions in [41]: (i) $\gamma = 0.826$ is too conservative for small topologies as the bound gap is negative in T-10 and T-15. (ii) As the number of nodes increases, $\gamma = 1.0$ and $\gamma = 0.826$ get looser and tighter, respectively.

7.2. Dynamic Demands Performance Bound

Dynamic demands performance bound problem is, in fact, the performance bound of the joint QoS routing and channel assignment problem in which each demand δ_i arrives at time t_i and has a limited holding time $e_i - t_i$. Again, the question is the maximum number of admissible demands. As explained before, for the problem, we first develop an upper bound model by extending RelaxedStatic; then, propose another model, which is equivalent to the first one and is decomposable; finally, we develop a decomposition algorithm that divides the second model into subproblems and solves them sequentially.

7.2.1. Upper Bound Model

As mentioned, in this problem, we should model the dynamics of the demands, which need to update network configurations (routing and channel assignment) at the demand arrival times. We deal with the problem by extending RelaxedStatic in the following ways. First, we introduce a time set T, which is $T = \{t_1, t_2, \dots, t_h\}$, and duplicate the channel assignment variables, $x^k_{(u,v)}$, for each $t_j \in T$, i.e., we add new variables $x^k_{(u,v),t_j}$ $\forall k \in K,\ \forall (u,v) \in E,\ \forall t_j \in T$. Second, we assume that accepted demands can be *rerouted*; thus, flow routes are time-dependent and reoptimized at each demand arrival time. They are denoted by $f^i_{(u,v),t_j}$ $\forall \delta_i \in \Delta,\ \forall (u,v) \in E,\ \forall t_j \in T$. Third, decision variable a_i is not duplicated because a demand is either accepted or not independent of the time we observe the network. Fourth, the required bandwidth of demand δ_i is defined as

$$b_{i,t_j} = \begin{cases} b_i, & \text{if } t_i \le t_j \le e_i \\ 0, & \text{otherwise.} \end{cases}$$

These extensions yield a model that is composed of h instances of the RelaxedStatic model, an instance per demand arrival. At each arrival time $t_j \in T$, decision variables must satisfy the constraints of the instance of RelaxedStatic corresponds to the time, which are defined as following.

Definition 6. *Constraints must be satisfied at time t_j,* ConsSet(t_j)*, are*

$$x^k_{(u,v),t_j} \le 1 \qquad \forall k \in K,\ \forall (u,v) \in E,$$

$$l^k_{(u,v),t_j} = x^k_{(u,v),t_j} c^k_{(u,v)} \qquad \forall k \in K,\ \forall (u,v) \in E,$$

$$\sum_{k \in K} \Big(\sum_{(u,v)\in E} x^k_{(u,v),t_j} + \sum_{(v,u)\in E} x^k_{(v,u),t_j} \Big) \le r_v \qquad \forall v \in V,$$

$$\sum_{(u,v)\in Q_i} \frac{l^k_{(u,v),t_j}}{c^k_{(u,v)}} = \sum_{(u,v)\in Q_i} x^k_{(u,v),t_j} \le \gamma \qquad \forall k \in K,\ \forall Q_i \in \Phi,$$

$$\sum_{\delta_i \in \Delta} f^i_{(u,v),t_j} = \sum_{k\in K} l^k_{(u,v),t_j} \qquad \forall (u,v) \in E,$$

$$\sum_{(u,v)\in E} f^i_{(u,v),t_j} - \sum_{(v,u)\in E} f^i_{(v,u),t_j} = \begin{cases} a_i b_{i,t_j}, & \textit{if } u = s_i \\ -a_i b_{i,t_j}, & \textit{if } u = d_i \quad \forall u \in V,\ \forall \delta_i \in \Delta, \\ 0, & \textit{otherwise} \end{cases} \tag{18}$$

and

$$x^k_{(u,v),t_j} \ge 0,\ a_i \in \{0,1\},\ l^k_{(u,v),t_j} \ge 0,\ f^i_{(u,v),t_j} \ge 0.$$

The major complexity of this model is that these instances are *not* independent because variables a_i $\forall \delta_i \in \Delta$ appear in all of them. In other words, demands are *not preemptable*; if a demand is accepted in the solution of one of the instances, it must be accepted in the remaining. As a result, we have to solve the h instances altogether simultaneously.

An important issue in modeling the dynamic demands performance problem, which needs to be addressed carefully, is the objective function of the model. If (3) is optimized, this model will be an appropriate model for the *off-line* joint QoS routing and channel assignment problem, in which the information about all demands is given at the beginning and solving the model finds the maximum number of admissible demands. However, in this paper, we have focused on the *on-line greedy CAC* strategy, where the information about a demand is not known before its arrival time and at demand arrival time, since the on-line algorithm does not aware of future demands, it greedily attempts to accept the given demand. We borrow the idea proposed in [43] to model this behavior of on-line algorithms, which is assigning profit ρ_i to demand δ_i and maximizing the aggregate profit of accepted demands. Suppose Δ is sorted in ascending order of t_i, the profit is assigned as

$$\rho_i = 2^{h-i}. \tag{19}$$

and the objective function is

$$\text{maximize} \sum_{\delta_i \in \Delta} a_i \rho_i. \tag{20}$$

These profits imply that if there is a feasible path for demand δ_i, it is not rejected in favor of accepting subsequent demands δ_j because $\rho_i > \sum_{j=i+1}^{h} \rho_j$ $\forall \delta_i \in \Delta$. This inequality implies that the model first puts its effort to accept δ_1, then consider δ_2, after that, δ_3 and so on; this exactly simulates the behavior of on-line greedy CAC algorithms.

The optimization model for the dynamic demands performance problem is

Model:	DynamicUB1$(\Delta, \boldsymbol{\rho})$
Objective:	(20)
Subject to:	ConsSet(t_j) $\quad \forall t_j \in T$.

Where $\boldsymbol{\rho}$ is the profit assignment vector obtained by (19). Since instead of OptimalStatic, we use the RelaxedStatic model, DynamicUB1 provides an upper bound on the network performance achievable through on-line greedy algorithms. Tightness of the model depends on the scale used in RelaxedStatic and as discussed in the previous subsection, in large networks, scale 0.826 yields a tighter bound than scaled 1.0.

There is a problem about DynamicUB1, this model will be huge even for medium size networks and a large number of demands; it is not solvable for practical networks. In the following, we decompose it to deal with this issue.

7.2.2. Decomposition

In this subsection, at the first step, we define a model that is equivalent to DynamicUB1, the set of accepted demands is the same for both models. Then, in the second step, we show this model is decomposable and develop an algorithm to decompose it.

The decomposable model is obtained through following modifications in DynamicUB1. First, instead of a_i for each demand we consider a_{i,t_j} for each demand δ_i and $t_j \in T$. Second, we consider two additional constraints

$$a_{i,t_j} \leq b_{i,t_j} M \qquad \forall \delta_i \in \Delta,\ \forall t_j \in T, \tag{21}$$

where $M > \big(\min\{b_i\}\big)^{-1}$ and

$$a_{i,t_j} \leq a_{i,t_i} \qquad \forall \delta_i \in \Delta,\ \forall t_j \in T. \tag{22}$$

Constraints (21) and (22) impose that a_{i,t_j} must be zero if $b_{i,t_j} = 0$ or $a_{i,t_i} = 0$, respectively. Third, profit assignment is

$$\rho_{i,t_j} = \begin{cases} 0, & t_j < t_i \\ 1, & t_j = t_i \\ 2, & t_j > t_i. \end{cases} \qquad \forall \delta_i \in \Delta,\ \forall t_j \in T. \tag{23}$$

Fourth, the new objective function is

$$\text{maximize} \sum_{t_j \in T} \sum_{\delta_i \in \Delta} a_{i,t_j} \rho_{i,t_j}. \tag{24}$$

Consequently, the decomposable model is

Model:	$\textsc{DynamicUB2}(\Delta, \boldsymbol{\rho})$
Objective:	(24)
Subject to:	(21), (22), and $\textsc{ConsSet}'(t_j) \quad \forall t_j \in T$.

Where $\textsc{ConsSet}'(t_j)$ is the same set of constraints denoted by $\textsc{ConsSet}(t_j)$ but a_i is replaced by a_{i,t_j} and $\boldsymbol{\rho}$ is the profit assignment vector obtained by (23).

We use following propositions to show that DynamicUB1 and DynamicUB2 are equivalent models.

Proposition 1. *In the solution of* DynamicUB2*, we have* $a_{i,t_i} = a_{i,t_j}$ *where* $t_i \leq t_j \leq e_i$.

Proof. The proof can be found in the Appendix. □

Proposition 2. *For a given* Δ*, we have* $a_i = a_{i,t_i}$*, where* a_i *and* a_{i,t_i} *are obtained by solving the* DynamicUB1 *and* DynamicUB2 *models, respectively.*

Proof. The proof can be found in the Appendix. □

In DynamicUB1, the accepted demands are determined by a_i, demand δ_i is accepted if $a_i = 1$; now, if we define it in DynamicUB2 using a_{i,t_i}, demand δ_i is accepted if $a_{i,t_i} = 1$, the set of accepted demands obtained by both models are the same due to proposition 2; in other words, these models are equivalent.

The distinguishing feature of the DynamicUB2 model is that we can decompose it into h subproblems and solve them sequentially. Note that constraint $\textsc{ConsSet}'(t_j)\ \forall t_j \in T$ in the model is indeed h independent sets of constraints, there is not any common variable among them. Each set contains the decision variables, and constraints correspond to a $t_j \in T$. Therefore, DynamicUB2 can be decomposed into h subproblems where jth subproblem is

Model:	$\textsc{subUB2}(\Delta'_{t_j}, \boldsymbol{\rho},\ j)$
Objective:	maximize $\sum_{\delta_i \in \Delta'_{t_j}} a_{i,t_j} \rho_{i,t_j}$
Subject to:	$\textsc{ConsSet}'(t_j)$.

In this model, constraints (21) and (22) are not included, we take them into account using $\Delta'_{t_j} \subset \Delta$ instead of Δ. Demand $\delta_i \notin \Delta'_{t_j}$ if either of these conditions holds

- $t_j < t_i$ or $t_j > e_i$ since $b_{i,t_j} = 0$ and constraint (21) implies $a_{i,t_j} = 0$.
- $t_i < t_j \leq e_i$ and $a_{i,t_i} = 0$ because constraint (22) enforces $a_{i,t_j} = 0$.

Since when $t_i < t_j \leq e_i$, constraint (21) does not impose any restriction, the first condition is equivalent to the constraint. In a similar way, the second condition is equivalent to (22) as the constraint is don't-care when $a_{i,t_i} = 1$. Therefore, subUB2 is a valid decomposition of DynamicUB2 as it takes all the constraints of DynamicUB2 into consideration.

Note that the second condition needs the optimal value of a_{i,t_i}; it implies that $\textsc{subUB2}(\Delta'_{t_i}, \boldsymbol{\rho},\ i)$ must be solved before $\textsc{subUB2}(\Delta'_{t_j}, \boldsymbol{\rho},\ j)$ for each $j > i$; in other words, subUB2 subproblems should be solved sequentially starting from $\textsc{subUB2}(\Delta'_{t_1}, \boldsymbol{\rho},\ 1)$. This is implemented by the decomposition algorithm as shown in algorithm 6. In this algorithm, if δ_i is rejected, it is removed from Δ'

Algorithm 6 : MOSTGREEDYONLINE(Δ)

```
1:  Create empty set $\Delta'$
2:  for $i = 1$ to $h$ do
3:      $\delta_i \leftarrow \Delta[i]$
4:      Add $\delta_i$ to $\Delta'$
5:      Assign profits $\boldsymbol{\rho}$ according (23)
6:      Solve SUBUB2($\Delta'$, $\boldsymbol{\rho}$, $i$)
7:      if $a_{i,t_i} = 1$ then
8:          Add $\delta_i$ to the Accepted demand set
9:      else
10:         Remove $\delta_i$ from $\Delta'$
11:     for each $\delta_j \in \Delta'$ do
12:         if $e_j < t_{i+1}$ then
13:             Remove $\delta_j$ from $\Delta'$
```

in line 10, due to (22); moreover, it is removed in line 13 because of (21) if it does not overlap with the next demand.

This algorithm is named "most greedy on-line" since it is on-line, it does not use the information of a demand before it arrives, and is the greediest algorithm; it accepts demands if there is a feasible network configuration, which is checked by solving the SUBUB2 optimization model.

8. Simulation Results

In this section, we present simulation results to evaluate the performance of the JQRCA algorithm. After clarifying the simulation setup and simulated algorithms, we study the effect of different parameters on the performance of the algorithms. Moreover, we present results on the average case computational complexity and overhead of the JQRCA algorithm.

8.1. Simulation Setup

We used a flow-level event-driven simulator developed in Java. Simulations were performed on an Intel Pentium IV 3.0 GHz machine with 2 Gigabytes RAM and CPLEX 11.0 was used. Three random topologies with 15, 25, and 50 nodes as shown in Table 6 were used. In each run of the simulations, a set of random traffic demands with the parameters shown in Table 8 was used. These were the default values used in all simulations, unless otherwise is stated. The simulation parameters, e.g., λ, B_{max}, and κ, were tuned to consider the networks under lightly loaded to highly overloaded conditions. The results presented in this section are the average obtained from ten different demand sets.

We simulated three solutions for the on-line joint QoS routing and channel assignment problem: the JQRCA algorithm, the MOSTGREEDYONLINE algorithm, and a static solution. In the static solution, at the beginning, we assign channels by the minimum interference greedy channel assignment algorithm [44], which is a static channel assignment to minimize total network interference, then, route flows using the minimum hop count routing. In the following figures, "MGO-0.82," "MGO-1.00," "JQRCA," and "Static" are the results of

Table 8: Parameters of simulation traffic

Parameter	Value
Number of demands (h)	300
Arrival rate	Poisson random variable with mean λ demands per min.
Holding time ($\mu_i^{-1} = e_i - t_i$)	Exponential random variable with mean 10 min.
Required bandwidth (b_i)	Uniform random variable in [1, B_{max}] Mb/s

Table 9: The correspondence between demand arrival rate, incoming traffic, and overload percentage

λ (demand/min)	Incoming Traffic (Mb/s)	Overload %		
		T-15	T-25	T-50
2	210	22	18	15
4	420	44	36	30
6	630	66	54	45
8	840	88	72	60
10	1050	110	90	75

MostGreeyOnline with $\gamma = 0.826$, MostGreeyOnline with $\gamma = 1.0$, the JQRCA algorithm with $k = 2$, and the static solution, respectively.

In the following results, we report the incoming traffic in terms of demand arrival rate. The incoming traffic can also be easily measured in terms of bandwidth as follows. The average holding time of demands is μ^{-1} minutes and λ new demands arrive in each minute; thus, using the Little's law, there are $\lambda\mu^{-1}$ traffic demands on average. Since the bandwidth requirement of demands is a uniform random variable in [1, B_{max}] Mb/s, its average value is $\frac{B_{max}+1}{2}$ Mb/s. Therefore, the total traffic offered to the network in terms of bandwidth is $0.5\lambda\mu^{-1}(B_{max} + 1)$ Mb/s. Table 9 shows the correspondence between demand arrival rate and incoming traffic. Moreover, this table shows the overload percentage of each topology per arrival rate. The overload percentage is the percentage of the incoming traffic over the maximum throughput of the network. The maximum throughput of the network is measured as follows. We load the network very heavily, e.g., $\lambda = 1000$ demand/min, use the MGO-1.00 to route the demands, and take the moving average of the bandwidth of the accepted demands over the time. The maximum throughput of the T-15, T-25, and T-50 topologies are 995 Mb/s, 1170 Mb/s, and 1394 Mb/s, respectively.

To measure the performance of QoS routing algorithms, we use the *demand acceptance rate* metric [11, 45]. Demand acceptance rate is the number accepted demands divided by the total number of demands. We discuss the relation between bandwidth of accepted demand and this metric, which depends on the fairness of the algorithm, in subsection 8.6.

8.2. Effect of Demand Arrival Rate

Arrival rate of demands is the first parameter we investigated. The acceptance rates of the solutions versus the demand arrival rate are shown in Fig. 5. These figures show that the JQRCA algorithm is an efficient algorithm, it significantly outperforms the static solution and has a comparable performance to the bound obtained by the MostGreedyOnline algorithm independent of the size of the network. As we see, in the worst case, the solution obtained by JQRCA is not far from the solution of MGO-0.82 more than 6–8%.

In Fig. 6, we depict the average solution time per SUBUB2 subproblem to show the efficiency of the MOSTGREEDYONLINE algorithm. This problem is solved by the algorithm at the arrival time of each demand. This figure shows that the algorithm is not time-consuming as the solution time is less than 0.6 second, 1.2 second, 12 second in the T-15, T-25, and T-50 topologies, respectively.

8.3. Effect of Maximum Required Bandwidth

The maximum required bandwidth, B_{max}, is a parameter influencing the offered load; hence, the performance of the algorithms depends on it. The acceptance rates of the algorithms versus the maximum required bandwidth are depicted in Fig. 7. These results show that acceptance rate is a decreasing function of B_{max} but the rate of reduction for JQRCA is much less than the Static solution and is comparable to the rate of MGO-0.82. As seen in the figures, the gap between the acceptance rates of JQRCA and MGO-0.82 enlarges as B_{max} increases; this is due to the flow-splitting and multipath routing. A flows that needs large amount of bandwidth is split into multiple sub-flows by the MOSTGREEDYONLINE algorithm, which are routed through multiple paths; however, the JQRCA algorithm is not allowed to split flows and cannot find a feasible path for the bandwidth intensive flows.

8.4. Effect of Number of Available Channels

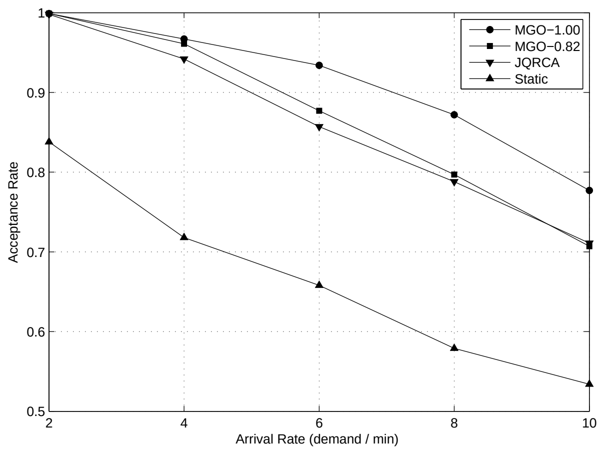
An efficient joint QoS routing and channel assignment algorithm should be able to exploit available channels. We conducted simulations with different numbers of channels in order to compare the algorithms from this point of view. In these simulations, λ is 4 demands per minute that leads to 420 Mb/s offered load. Fig. 8 shows the performance of the algorithms versus the number of available channels. In all topologies, JQRCA exploits the available channels as well as the MOSTGREEDYONLINE algorithm since the rate of increasing of acceptance rate is almost the same for both algorithms. In the T-15 topology, Fig. 8(a), JQRCA outperforms MGO-0.82, this confirms our previous results in Section 7.1.3 that showed the scale 0.826 is too conservative in small topologies.

8.5. Effect of Number of Radios per Node

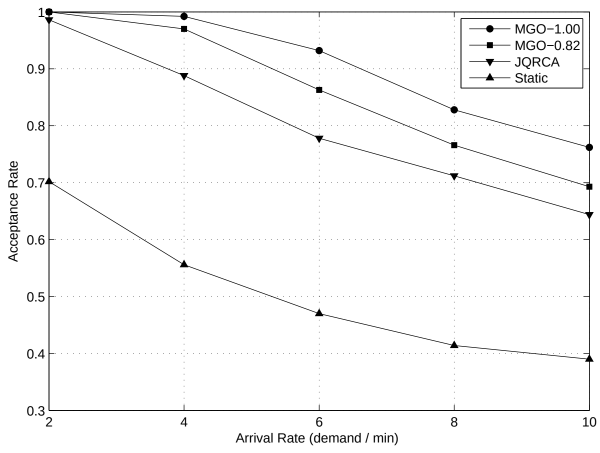
Radios in each node are scarce resources. An efficient algorithm should be capable of providing good performance even using a limited number of radios. We show the performance of the algorithm versus the numbers of radios per node in Fig. 9. These figures show that JQRCA does not need many radios per node to achieve high network performance.

When there are two or three radios per node, there is a large gap between JQRCA and MGO-0.82. This is due to the infeasibility of the solutions of the SUBUB2 model, which is explained in Section 7.1.2. The unschedulability issue is more serious when r_u is small; so, in this case the MOSTGREEDYONLINE algorithm gives a bit loose upper bound on the network performance.

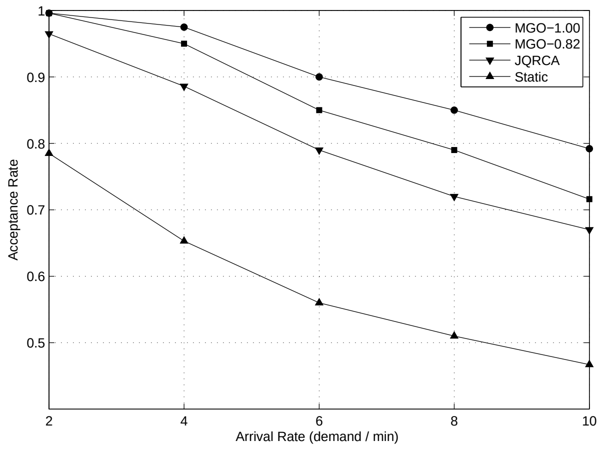
Fig. 8 and Fig. 9 indicate the main factor limiting the network performance is the number channel not the number of radios. In real-life WMNs, in which each node has three/four radios, JQRCA can exploit the many existing channels, e.g., 12 orthogonal channels in IEEE 802.11a, to obtain a near optimal network performance.



(a) T-15 Topology

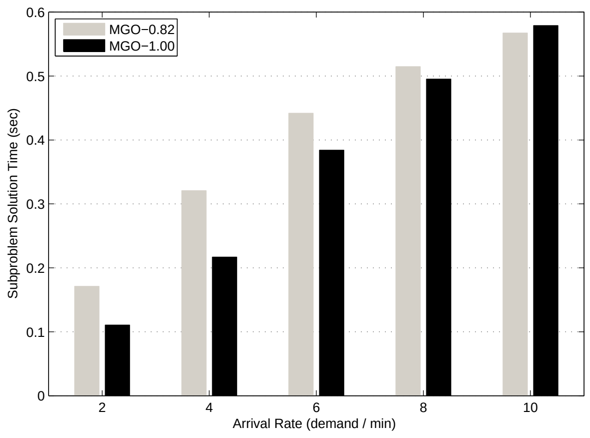


(b) T-25 Topology

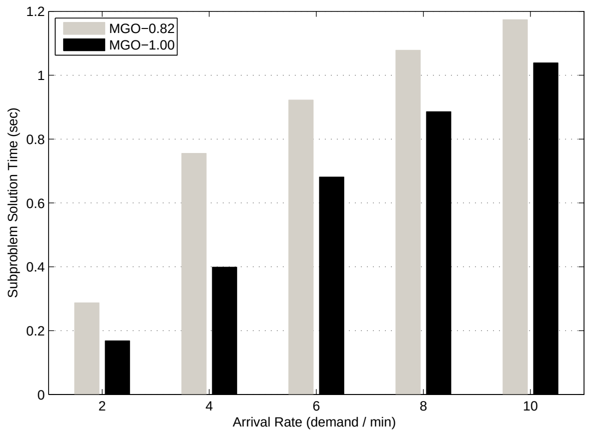


(c) T-50 Topology

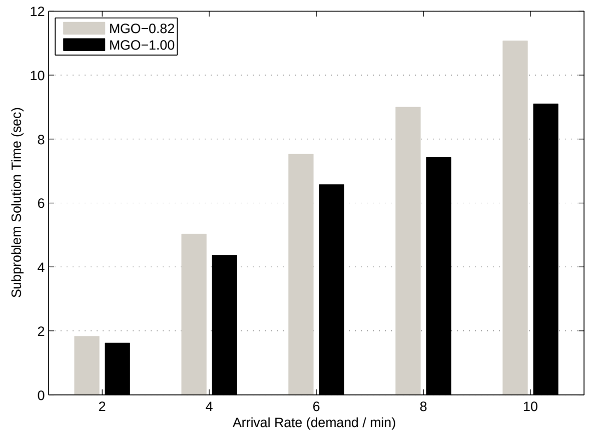
Figure 5: Acceptance rate versus demand arrival rate. The parameters of the topologies and the simulation traffic are shown in tables 6 and 8, respectively; and $B_{max} = 20$.



(a) T-15 Topology

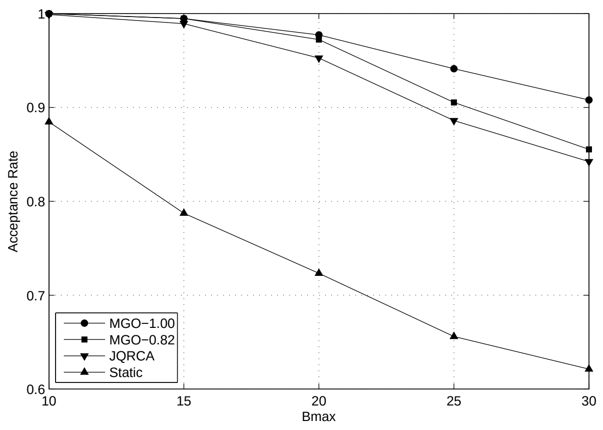


(b) T-25 Topology

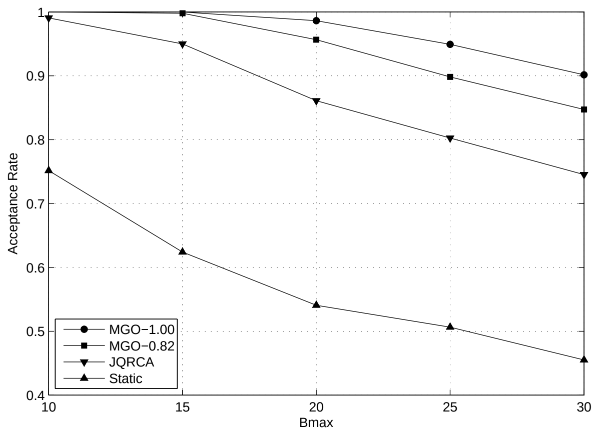


(c) T-50 Topology

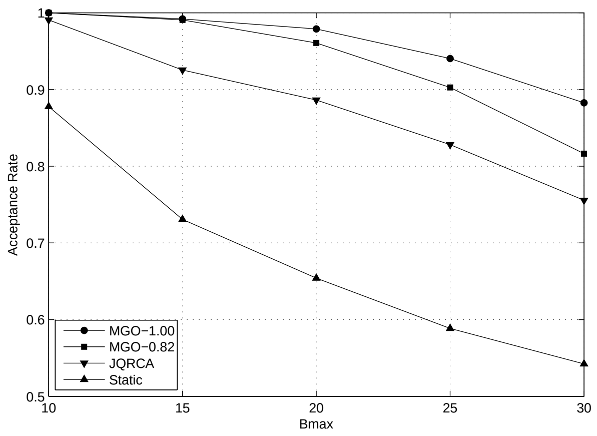
Figure 6: Average solution time of subUB2 versus demand arrival rate. The parameters of the topologies and the simulation traffic are shown in tables 6 and 8, respectively; and $B_{max} = 20$.



(a) T-15 Topology

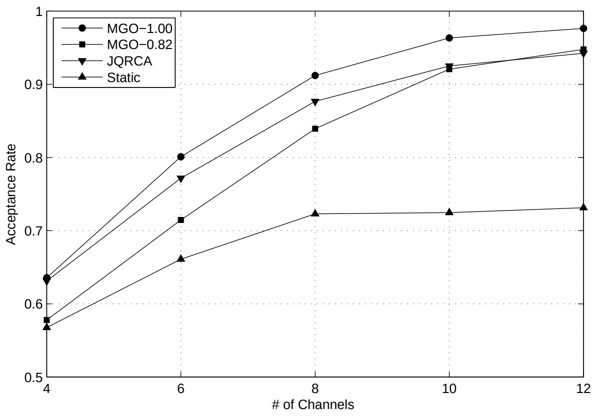


(b) T-25 Topology

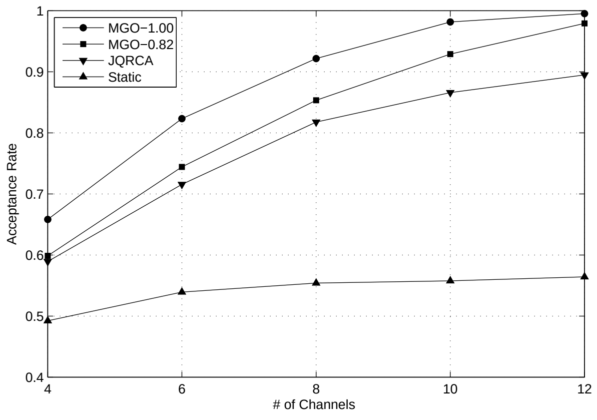


(c) T-50 Topology

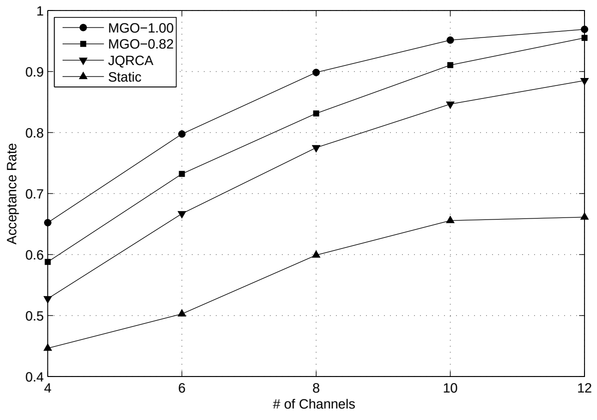
Figure 7: Acceptance rate versus the maximum required bandwidth. The parameters of the topologies and the simulation traffic are shown in tables 6 and 8, respectively; and $\lambda = 4$ demands per minutes.



(a) T-15 Topology

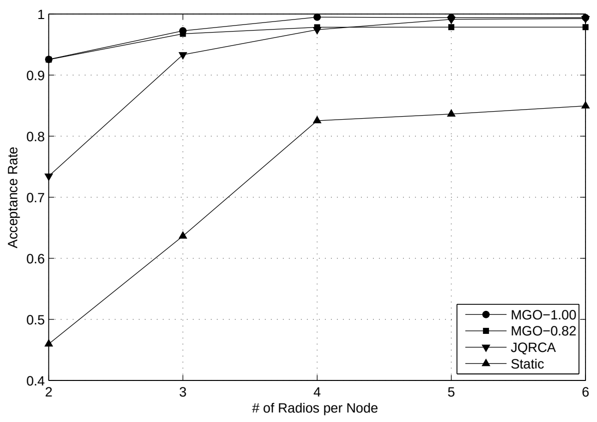


(b) T-25 Topology

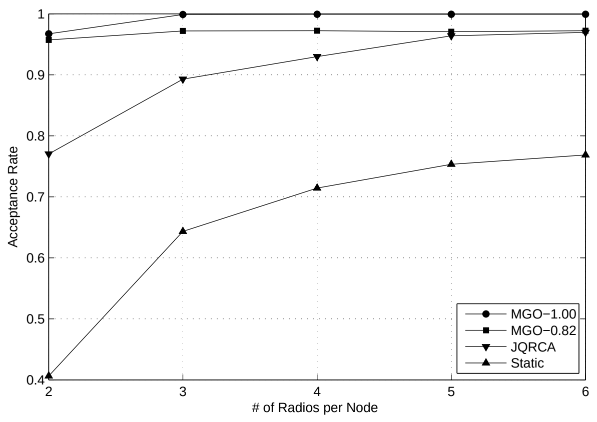


(c) T-50 Topology

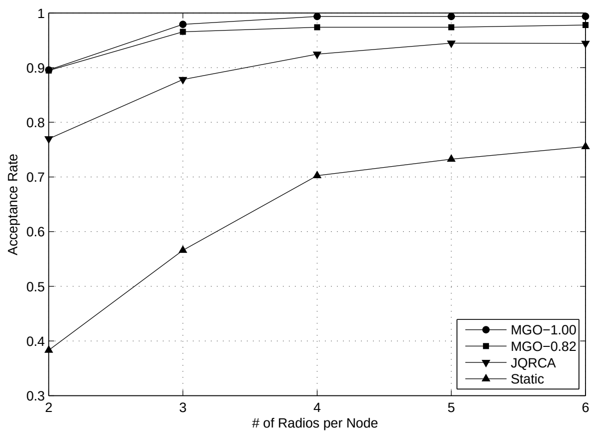
Figure 8: Acceptance rate versus the number of channels. The parameters of the topologies and the simulation traffic are shown in tables 6 and 8, respectively. $\lambda = 4$ demands per minute and $B_{max} = 20$.



(a) T-15 Topology



(b) T-25 Topology



(c) T-50 Topology

Figure 9: Acceptance rate versus the number of radios per node. The parameters of the topologies and the simulation traffic are shown in tables 6 and 8, respectively. $\lambda = 4$ demands per minute and $B_{max} = 20$.

8.6. Fairness

As mentioned in Section 3.5, greedy CAC strategy is suitable to achieve (absolute) fairness. In this subsection, we measure the fairness of the JQRCA algorithm through the Jain's fairness index [46]. Suppose there are N traffic classes and $\mathcal{A}_i$ demands are accepted from class i. Fairness index is defined as

$$\text{Fairness index} = \frac{\left(\sum_{i=1}^{N} \mathcal{A}_i\right)^2}{N \sum_{i=i}^{N} \mathcal{A}_i^2}.$$

This parameter measures how fairly an algorithm accepts demands from different classes. *Absolute fairness* is achieved when the fairness index is equal to one, which implies the same number of demands is accepted from all classes; *absolute unfairness*, when all the accepted demands belong to only one class, is the case that fairness index is $\frac{1}{N}$.

Here, we define traffic classes based on the bandwidth requirements; we divide $[1,\ B_{max}]$ into 10 subintervals. Demand δ_i is in class j if $(b_i - 1) \in [(j-1)\frac{B_{max}}{10}, j\frac{B_{max}}{10}]$ Mb/s. The fairness index of JQRCA in various settings of network and traffic parameters is shown in Table 10. Note that these parameter settings are the values we used in the previous subsections. This table shows that JQRCA is quite fair, fairness index is more than 0.90 in most cases even when acceptance rate is about 0.6. These results confirm the intuition; since JQRCA attempts to accept every demand regardless of its bandwidth requirement, it achieves the absolute fairness in lightly or moderately loaded networks.

These results can be used to compute network throughput in terms of bandwidth. Let us denote the acceptance rate, average bandwidth requirement of accepted demands, and fairness index by α, $\overline{b}$ and η, respectively. The rate of admitted demands is $\alpha\lambda$ demand per minute. According to the Little's law, the average number of demands active in the network is $\alpha\lambda\mu^{-1}$. Consequently, the network throughput is $\alpha\lambda\mu^{-1}\overline{b}$ Mb/s. If JQRCA was absolutely fair, $\eta = 1$, it would accept exactly the same number of demands from each class and thus $\overline{b} = \frac{B_{max}+1}{2}$. However, JQRCA is a bit unfair in some cases. In these cases, $\overline{b} < \frac{B_{max}+1}{2}$ because the acceptance probability of the demands with small bandwidth requirements is more than the probability of demands that need large bandwidth. We take this observation into account through scaling down $\overline{b}$ by the fairness index; we approximate the network throughput as

$$\text{Approximated Throughput} = 0.5\alpha\lambda\mu^{-1}\eta^{\sigma}(B_{max}+1)\text{Mb/s},$$

where σ is a parameter to control the scaling. Our simulations show that this approximation is very accurate. In Table 10, column "Throughput" is the actual network throughput measured from the simulations. The last column in this table represents the throughput approximation normalized error. It is obtained by computing the difference between the actual network throughput and the approximated throughput and then dividing the difference by the actual throughput. We used $\sigma = 2$ in our simulations and as seen in the table, the error is almost less than 2.5%.

8.7. Complexity and Overhead

In subsections 5.4 and 6.4, we analyzed the worst case complexity of the QDDCA and JQRCA algorithms. In this section, we provide an insight into the

Table 10: Fairness index of JQRCA and throughput approximation normalized error.

Configuration					Throughput	Acceptance	Fairness	Approx.
Topology	λ	B_{max}	κ	r_u	(Mb/s)	Rate	Index	Error %
T-50	4	20	12	Random	347.94	0.858	0.990	2.1
T-25	4	20	12	Random	347.97	0.861	0.990	2.0
T-15	10	20	12	Random	568.81	0.646	0.922	1.9
T-50	10	20	12	Random	537.80	0.615	0.918	1.7
T-25	4	10	12	Random	220.19	0.992	0.998	1.9
T-15	4	10	12	Random	220.31	0.998	0.999	2.0
T-50	4	30	12	Random	412.12	0.736	0.963	2.9
T-25	4	30	12	Random	400.42	0.721	0.955	2.2
T-15	4	20	4	Random	211.34	0.603	0.907	2.2
T-50	4	20	4	Random	166.01	0.506	0.858	5.1
T-50	4	20	12	3	346.41	0.853	0.983	1.8
T-25	4	20	12	3	360.64	0.882	0.993	1.5
T-15	4	20	12	6	403.52	0.990	0.999	1.5
T-50	4	20	12	6	377.06	0.919	0.995	1.6

average case complexity and overhead of the algorithms through simulations.

The computational complexity of JQRCA is proportional to the number of violated links, which are generated during the search for a feasible path, and the average number of calls of LinkChannelChange to resolve a violated link. These statistics are presented in Table 11. In this table, column "Zero" refers to the violated links that are assigned to the virtual channel 0 and column "Non-Zero" are the remaining. When demand arrival rate is low, e.g., $\lambda = 2$, a large percentage of the violated links, e.g., 89% in T-15 or 91% in T-50, are the links on channel 0; because in this case, there are very few flows in the network and consequently, many links are assigned to the virtual channel since no load is on them. On the other hand, in high demand arrival rates, e.g., $\lambda = 10$, there are many flows in the network and a large number of links are assigned to a non-zero channel; therefore, a small percentage of the violated links, e.g., 19% in T-25 or 26% in T-50, are on channel 0. Table 11 also shows that LinkChannelChange is called more than one time per violated link. This is because GroupChannelChange generates new violations that LinkChannelChange is called for them too. However, this extra complexity is not more than one additional LinkChannelChange call, which is much less than what we considered in the worst case complexity analysis, $\hat{I}$.

After finding a feasible path, network is updated through messages sent from the call admission control server to the nodes in the network; the update includes the path establishment and updating channel assignment. The messaging overhead of the former and latter updates are respectively proportional to the average path hop count and the number of channel updates per accepted demand, which are reported in Table 12. It is seen that the number channel updates depends on the topology and the path hop count is much less than n. It is important to note that the overhead caused by the dynamic channel reassignment is not significant in comparison to the overhead of QoS routing; in fact, in the worst case, $\lambda = 8$ in T-50, the overhead of channel reassignment is less than 53% of the overhead of routing.

Tables 11 and 12 show that the complexity and overhead of the JQRCA algorithm in practice are much less than the worst case presented in Sections 5.4 and 6.4; we believe the algorithm is an efficient solution for real-life WMNs.

Table 11: Average case computational complexity of the JQRCA algorithm. The parameters of the simulation topologies and traffic are shown in tables 6 and 8, respectively. $B_{max} = 20$ and $k = 2$.

λ	Violations per Demand						LinkChannelChange Call per Violation		
	Zero			Non-Zero					
	T-15	T-25	T-50	T-15	T-25	T-50	T-15	T-25	T-50
2	0.87	1.24	1.88	0.10	0.22	0.17	1.03	1.04	1.019
4	0.57	0.88	1.38	0.68	1.15	1.13	1.44	1.19	1.40
6	0.46	0.78	1.21	1.34	1.84	1.75	1.56	1.26	1.51
8	0.43	0.71	1.189	1.84	2.41	2.82	1.70	1.27	1.68
10	0.40	0.68	1.11	2.26	2.79	3.06	1.84	1.33	1.73

Table 12: Overhead of the JQRCA algorithm. The parameters of the simulation topologies and traffic are shown in tables 6 and 8, respectively. $B_{max} = 20$ and $k = 2$.

λ	Channel Updates per Accepted Demand			Path Hop Count		
	T-15	T-25	T-50	T-15	T-25	T-50
2	0.882	1.303	1.990	2.096	2.871	4.090
4	0.828	1.106	1.850	2.053	2.814	4.023
6	0.848	1.086	1.942	2.021	2.832	3.952
8	0.891	1.108	2.040	2.001	2.747	3.871
10	0.882	1.072	1.933	1.922	2.696	3.840

9. Conclusions and Future Work

We studied the problem of performance optimization of multi-channel multi-radio WMNs in presence of traffic with QoS constraints, which is measured in terms of acceptance rate of traffic demands. To boost the network performance, we proposed an on-line joint QoS routing and channel (re)assignment algorithm that utilizes network resources efficiently by optimal routing and adapts them through appropriate channel reassignments. The algorithm does not require any prior information about the offered load, and aims to keep the number of channel reassignments small. At demand arrival times, the algorithm finds a path, if it is not feasible, it detects the violated links and attempts to resolve them. If all the violations are not resolved, another path is found, and so on. The algorithm examines up to k paths.

We approached the problem of finding an upper bound on the maximum number of admissible demands by formulating it as a MILP model and developing a decomposition algorithm for the model. It is important to note that the decomposition technique can be used for other similar problems e.g., performance bound of dynamic QoS routing in wired networks. Comparing the JQRCA algorithm to the bound obtained by the decomposition algorithm shows that in spite of the fact that JQRCA is not allowed to reroute existing flows, cannot use the flow splitting and multipath routing mechanisms, and is restricted to change only the channels in range $2I_R$ of flow routes, its performance is near to the bound in different network and traffic parameter settings. It can efficiently exploit available channels even with very few radios per node.

In this paper, we considered greedy CAC strategy, assumed perfect orthogonal channels, and proposed a centralized algorithm. We plan to develop a distributed version of JQRCA and extend it to consider non-greedy CAC mechanisms and adjacent channel interferences in the future. Moreover, routing metrics other than $|I_{(u,v)}|$, e.g., hop-count bounded widest path, can be considered in the future work.

10. Acknowledgment

We would like to thank the reviewers for their helpful comments.

Appendix A. Proof of Proposition 1 and 2

In the DynamicUB2 model, let X_{t_j} be a set contains demand δ_j and active demands at time t_j, demands that $a_{i,t_{j-1}} = 1$ and $b_{i,t_j} \neq 0$. Let $Y_{t_j} \subseteq X_{t_j}$ be the set of demands that satisfy $\textsc{ConsSet}'(t_j)$, and let $Z_{t_j} \subset Y_{t_j}$ be the set of demands that leave the network before t_{j+1}.

We prove proposition 1 as follows.

Proof. If $a_{i,t_i} = 0$, constraint (22) enforces that $a_{i,t_j} = a_{i,t_i} = 0 \; \forall t_i \leq t_j \leq e_i$. In the case of $a_{i,t_i} = 1$, we prove it by induction.

Base case: If $j = i + 1$, then we have $X_{t_j} = Y_{t_i} \cup \delta_j \setminus Z_{t_j}$. For the sake of simplicity of presentation, we assume $Z_{t_j} = \emptyset$, the proof is in a similar way when $Z_{t_j} \neq \emptyset$. If there is a feasible network configuration to accept all demands belong to X_{t_j}, we have $a_{i,t_j} = 1$, thus $a_{i,t_i} = a_{i,t_j} = 1$. Otherwise, if all the demands cannot be accepted, δ_j must be rejected because $\rho_{j,t_j} < \rho_{i,t_j} \; \forall \delta_i \in X_{t_j} \setminus \delta_j$. Rejecting δ_j is sufficient since there is a feasible configuration for $X_{t_j} \setminus \delta_j = Y_{t_j}$, thus in this case, also, $a_{i,t_i} = a_{i,t_j} = 1$.

Induction step: Suppose $j > i + 1$. If $e_i > t_j$, demand δ_i is an active demand, $\delta_i \in X_{t_j}$. According to the induction assumption, we have $a_{i,t_{j-1}} = 1$. Similar to the discussion about the base case, if all demands belong X_{t_j} are accepted, $X_{t_j} = Y_{t_j}$, we have $a_{i,t_j} = a_{i,t_{j-1}} = a_{i,t_i} = 1$; otherwise, again δ_j must be rejected, and it is sufficient since there is a feasible configuration for $Y_{t_{j-1}}$; therefore, again we have $a_{i,t_j} = a_{i,t_{j-1}} = a_{i,t_i} = 1$ that completes the proof.

□

The proof of proposition 2 is as follows.

Proof. We prove it by induction which is based on the number of demands, h.

Base case: When $h = 1$, $\Delta = \{\delta_1\}$, both DynamicUB1 and DynamicUB2 models are the same and if there is a feasible path for the demand, it is accepted, $a_1 = a_{1,t_1} = 1$, otherwise it is rejected, $a_1 = a_{1,t_1} = 0$.

Induction step: Suppose $a_i = a_{i,t_i}$ for $i = 1, \ldots, h$ when $h = j-1$. Increasing the number of demands to $h = j$ has two effects:

- It adds new constraint (18) corresponds to each δ_j to $\textsc{ConsSet}(t_i)$ and $\textsc{ConsSet}'(t_i)$ where $i = 1, \ldots, j-1$.
- It adds a new set of variables and constraints to both models, which are denoted by $\textsc{ConsSet}(t_j)$ and $\textsc{ConsSet}'(t_j)$ in DynamicUB1 and DynamicUB2, respectively.

We make the following observations

1. We neglect the first effect because for $i = 1, \ldots, j-1$, we have $b_{j,t_i} = 0$ and therefore, the right-hand side of (18) is 0 and the constraint is don't-care.

2. Adding ConsSet(t_j) to DynamicUB1 does not affect the optimal value of a_i for $i = 1, \ldots, j-1$ since $\rho_i > \rho_j$.
3. Adding ConsSet$'(t_j)$ to DynamicUB2 does not affect the optimal value of a_{i,t_i} for $i = 1, \ldots, j-1$ since the variable does not appear in ConsSet$'(t_j)$.

The first and second observations imply that the optimal value of a_i for $i = 1, \ldots, j-1$ obtained in the case of $h = j$ is equal to the values when $h = j-1$. The first and third observation imply the same conclusion for a_{i,t_i}. Therefore, when $h = j$, we have $a_i = a_{i,t_i}$ for $i = 1 \ldots j-1$; we only need to show $a_j = a_{j,t_j}$ to complete the proof. It is straightforward because the optimal value of a_j and a_{j,t_j} are determined by satisfaction of constraints ConsSet(t_j) and ConsSet$'(t_j)$, respectively. These satisfactions are influenced by the optimal values of a_i and a_{i,t_j} for $i = 1, \ldots, j-1$ and we know that $a_i = a_{i,t_i} = a_{i,t_j}$, where the last equality is due to proposition 1. Therefore, since $a_i = a_{i,t_j}$ and ConsSet(t_j) and ConsSet$'(t_j)$ are identical set of constraints, we have $a_j = a_{j,t_j}$ that completes the proof. $\square$

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