

# A general call admission policy for next generation wireless networks

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## Abstract

In this paper, we consider the call admission problem in cellular networks that support several classes of calls. In the first part of this paper, we first introduce a multi-threshold guard channel policy and study its limiting behavior under the stationary traffic. Then we give an algorithm for finding the optimal number of guard channels that minimizes the blocking probability of calls with lowest level of QoS subject to constraints on blocking probabilities of other calls. In the second part of the paper, we give an algorithm for finding the minimum number of channels subject to constraints on blocking probabilities of calls. Finally, we propose a prioritized channel assignment algorithm for multi-cells cellular networks to minimize the blocking probability of calls with lowest level of QoS subject to constraints on the blocking probabilities of other calls.

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## 1. Introduction

During the last decade, there has been a rapid growth of wireless communication technology. The next generation wireless networks are expected to eventually carry multi-media traffic such as voice only, mixed voice and data, image transmission, WWW browsing, email, etc. In order to support such wide range of traffic on a network, the network must be capable of satisfying various quality of service (QoS) requirements. The satisfying QoS means that various traffic should get predictable service from the available resources in the network. Since wireless spectrum remains as the prime limited resource in the next generation networks, it is necessary to develop mechanisms that can provide effective bandwidth management while satisfying the QoS requirement for incoming calls. In order to satisfy the QoS requirements, call admission control is needed.

The call admission policies determine whether a call should be either accepted or rejected at the base station and assign the required channel(s) to the admitted call. This results in a distributed call admission strategy which can be applied to every base station. In order to satisfy the level of QoS for each type of call, call admission control policies usually use the prioritized methods. This priority is usually implemented through allocation of more resources to calls with higher level of QoS. Several call admission control policies have been proposed to reduce the dropping of voice calls in wireless networks [1–5]. However, little attention is paid to wireless multi-media networks. In what follows, we review some call admission policies for cellular networks.

The simplest call admission policy is called *guard channel* policy (GC) [1]. Suppose that the given cell has  $C$  full duplex channels. The guard channel policy reserves a subset of channels, called *guard channels*, allocated to the cell for sole use of handoff calls (say  $C-T$  channels). Whenever the channel occupancy exceeds a certain threshold,  $T$ , the guard channel policy rejects new calls until the channel occupancy goes below the threshold. The guard channel policy accepts handoff calls as long as channels are available. It has been shown that

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there is an optimal threshold  $T^*$  in which the blocking probability of new calls is minimized subject to the constraint on the dropping probability of handoff calls [3]. Algorithms for finding the optimal number of guard channels are given in [2–4]. If only the dropping probability of handoff calls is considered, the guard channel policy gives very good performance, but the blocking probability of new calls is degraded to a great extent. In order to have more control on both the dropping probability of handoff calls and the blocking probability of new calls, *limited fractional guard channel policy* (LFG) is proposed [3]. This policy reserves a non-integral number of guard channels for handoff calls. The limited fractional guard channel policy uses an additional parameter  $\pi$  and operates the same as the guard channel policy except when  $T$  channels are occupied in the cell, in which case new calls are accepted with probability  $\pi$ . It has been shown that there is an optimal pair  $(T^*, \pi^*)$ , which minimizes the blocking probability of new calls subject to the constraint on the dropping probability of handoff calls [3]. An algorithm for finding such optimal parameters is given in [3]. *Uniform fractional channel policy* (UFC) is introduced in [6], which accepts new calls with probability of  $\pi$  independent of channel occupancy. It is shown that there is an optimal  $\pi^*$ , which minimizes the blocking probability of new calls subject to the constraint on the dropping probability of handoff calls. An algorithm for finding such optimal parameter is given in [5]. Then conditions under which the uniform fractional channel policy performs better than the guard channel policy is derived. It is concluded that, the uniform fractional channel policy performs better than the guard channel policy under low handoff traffic conditions.

In [7,8], a cellular network with two traffic classes is considered and the call admission problem is formulated as a semi-Markov decision process problem. Since, it is too complex to have a closed form solution for this semi-Markov decision process, Q-learning [7] and neuro-dynamic programming [8] are used. In [9], learning automata are used for deciding to admit/reject new calls and in [10], learning automata are used for determination of the number of guard channels. Adaptive version of LFG is also introduced in [11]. In [12], two traffic classes of voice and transactions are considered and static and dynamic guard channel schemes are proposed to maintain an upper bound on dropping probability of handoff transaction calls. In this approach,  $(C-T)$  guard channels are reserved for handoff transaction calls, but new calls and handoff voice calls have the same priority. Thus, this scheme cannot be used to maintain the QoS for the handoff voice calls.

All of the above mentioned call admission policies consider only one threshold to decide for accepting/rejecting of new calls. These policies cannot be used when there is several classes of traffic with different level of QoS. In such

cases, we need multi-threshold scheme, which provides different thresholds for different classes.

*Dual-threshold reservation* (DTR) scheme for integrated voice/data wireless networks is given in [13,14]. In this scheme, three classes of calls in ascending order of level of QoS are considered which are data calls (both new and handoff calls), new voice calls and handoff voice calls. The basic idea behind the DTR scheme is to use two thresholds, one for reserving channels for handoff voice, while the other is used to block data traffic into the network in order to preserve the voice performance in terms of handoff call dropping and new call blocking probabilities. In [13,14], DTR scheme is modeled using a two-dimensional Markov chain and the effect of different values for number of guard channels on dropping and blocking probabilities are plotted, but no algorithm for finding the optimal number of guard channels is given. *Two-threshold guard channel* (TTGC) scheme for wireless networks handling two classes of voice users is introduced in [15]. In this scheme, three classes of traffic in ascending order of level of QoS are considered which are new calls, handoff voice calls of calls 1 and class 2. In TTGC scheme two thresholds are used to provide the level of QoS for different classes. The limiting behavior of TTGC scheme is analyzed under stationary traffic. The TTGC scheme minimizes the blocking probability of new calls subject to the constraint on the dropping probabilities of calls of two classes. In [15], three other problems are also considered: finding the optimal number of guard channels, minimizing number of required channels and the optimal channel assignment in multi-cells cellular networks while satisfying the level of QoS. In order to solve these problems, three optimal algorithms are introduced. For a comprehensive survey of call admission algorithms, interested readers may refer to [16].

In this paper, the idea given in [15] is generalized to multi-classes, where there are  $N$  classes of traffic each of which having different level of QoS. Each class may contain either new or handoff calls but having the same level of QoS. Our proposed model, referred to as *multi-threshold guard channel* (MTGC) scheme, builds upon guard channel scheme by using  $N-1$  thresholds in which each threshold is used to reserve channels for satisfying the specified level of QoS for that class. The limiting behavior of MTGC is analyzed under stationary traffic using one dimensional Markov chain. In this paper, we also consider three problems: finding the optimal thresholds, minimizing the number of required channels, and the optimal channel assignment in multi-cells cellular networks while satisfying the level of QoS.

The rest of this paper is organized as follows: Section 2 presents the multi-threshold guard channel policy. Section 3 gives an algorithm to find the optimal value of thresholds and Section 4 gives an algorithm for finding the minimum numbers of channels required to maintain the pro-specified level of QoS. Section 5, presents an optimal channel

assignment algorithm for multi-cells cellular networks and Section 6 concludes the paper.

## 2. Multi-threshold guard channel scheme

In this section, the idea of two-threshold guard channel scheme is generalized to multi-classes and *multi-threshold guard channel scheme* is introduced. In this section, we first introduce a multi-threshold guard channel scheme for a cellular network with  $N$  classes of calls and then compute its blocking performance. We consider a homogenous cellular network where all cells have the same number of channels  $C$  and experience the same call arrival rates for all types of calls. In each cell, the arrival of calls of class  $k$  ( $k=1, \dots, N$ ) is Poisson distributed with arrival rate  $\lambda_k$  and the channel holding time of calls of class  $k$  ( $k=1, \dots, N$ ) is exponentially distributed with mean  $\mu_k^{-1}$ . Thus, the total call arrival rate is  $\Lambda_0 = \lambda_1 + \dots + \lambda_N$ . These assumptions have been found reasonable as long as the number of mobile users in a cell is much greater than the number of channels allocated to that cell. Assume that the calls of class  $k$  (for  $k=1, \dots, N$ ) have a certain level of QoS such that its blocking probability must be less than  $q_k$ . Without loss of generality, it is assumed that  $q_1 \geq q_2 \geq \dots \geq q_N$ . This implies that calls for class  $k$  (for  $k=1, \dots, N-1$ ) require less resources than calls of class  $k+1$ , i.e. calls for class  $k+1$  have a higher priority than calls of class  $k$ .

To provide the specific level of QoS for calls, the allocated channels of each cell are partitioned into  $N$  subsets. In order to partition the channel sets,  $(N-1)$  thresholds,  $T_1, \dots, T_{N-1}$  ( $0 < T_1 \leq T_2 \leq \dots \leq T_{N-1}$ ) are used. For the sake of simplicity in presentation, we use two additional fixed thresholds  $T_0 = -1$  and  $T_N = C$ . The procedure for accepting calls in multi-threshold guard channel scheme (Algorithm 1) can be described as follows. A call from class  $k$  (for  $k=1, \dots, N$ ) is accepted when the number of busy channels is smaller than  $T_k$ ; otherwise the call is blocked.

**Algorithm 1.** Multi-threshold guard channel scheme

```

loop
  if Call of Class  $k$  then
    if  $c(t) < T_k$  then
      accept call
    else
      reject call
    end if
  end if
end loop
    
```

If the cell is in statistical equilibrium, then it can be modeled as a  $N$ -dimensional Markov chain with the following state space.

$$S = \left\{ (o_1, o_2, \dots, o_N) \mid o_1 \geq 0, \dots, o_N \geq 0, \text{ and } \sum_{k=1}^N o_k \leq C \right\}, \quad (1)$$

where  $o_k$  (for  $k=1, \dots, N$ ) denotes the number of calls of class  $k$  in the cell. Let  $e_j$  be a  $(N+1)$ -dimensional unit vector with 1 as the  $j$ th element and the rest zero,  $q = (o_1, o_2, \dots, o_N)$  be the state of the cell and  $q(q; \tilde{q})$  be the state transition rate from state  $q$  to state  $\tilde{q}$ . Then for all  $j=1, 2, \dots, N$ , we have

$$\begin{aligned} q(q; q + e_j) &= \lambda_j && \text{if } \sum_{k=1}^N o_k < T_j \\ q(q + e_j; q) &= (o_j + 1)\mu_k && \text{if } \sum_{k=1}^N o_k < C. \end{aligned} \quad (2)$$

From the above transition rates, the balance equation of the Markov chain can be written. Let  $p(q)$  be the steady state probability that there is  $o_k$  (for  $k=1, \dots, N$ ) calls of class  $k$  in a cell. Let  $T$  and  $B_k(T)$  (for  $k=1, \dots, N$ ) be the threshold set  $[T_0, T_1, \dots, T_N]$  and the blocking probability of calls for class  $k$  when the cell uses threshold set  $T$ , respectively. Thus, the blocking probability of class  $k$  is equal to

$$B_k(T) = \sum_{q \in S_k} p(q), \quad (3)$$

where

$$S_k = \left\{ (o_1, o_2, \dots, o_N) \mid o_1 \geq 0, \dots, o_N \geq 0, \text{ and } \sum_{j=1}^N o_j \geq T_k \right\}. \quad (4)$$

In order to calculate the above equations, we need to calculate the steady state probabilities  $p(q)$ , which can be obtained using a method given in [17]. This technique is based on typical feature of Chapman–Kolomorgoff system of equations in which there exist a subset of states, called *boundary states*, for which all other state probabilities can be expressed as a function of state probabilities of the boundary states. The basic idea of this technique is to choose the boundary states first to derive the expressions for all remaining state probabilities and then solve a reduced system of equations for these boundaries. Then all state probabilities can be determined by means of the boundary states. Since, this technique does not lead to a suitable closed form solution for all Markov chains including the  $N$ -dimensional Markov chain for the proposed model, we make the following assumption regarding the traffic parameters to reduce the dimensionality of the Markov chain to one.

**Assumption 1.** The channel holding times for all types of calls are exponentially distributed with the same mean  $\mu^{-1}$ , i.e.  $\mu = \mu_1 = \mu_2 = \dots = \mu_N$ .

Let  $c(t)$  denote the number of occupied channels in the given cell and

$$\Lambda_k = \sum_{j=k+1}^N \lambda_j,$$

$\alpha_k = \Lambda_k / \Lambda_0$  and  $\rho = \Lambda_0 / \mu$ . In the multi-threshold guard channel scheme,  $\{c(t) \mid t \geq 0\}$  is a continuous-time Markov

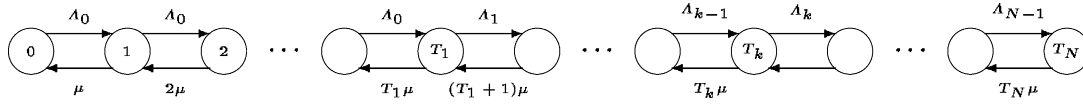


Fig. 1. Markov chain model of a cell for multi-threshold guard channel scheme

chain (birth–death process) with states  $\{0,1,\dots,C\}$ . The state transition diagram of a cell with  $C$  channels, which uses multi-threshold guard channel scheme, is shown in Fig. 1.

From the structure of the Markov chain, we can easily write down the steady-state balance equations [18]. Define the steady state probability

$$P_n = \lim_{t \rightarrow \infty} \text{Prob}[c(t) = n] \quad n = 0, 1, \dots, T_N. \quad (5)$$

By writing down the equilibrium equations for the steady-state probabilities  $P_n$  ( $n=0,1,\dots,T_N$ ), we obtain

$$A_k P_{n-1} = n \mu P_n \quad T_k \leq n \leq T_{k+1}.$$

Solving the above balance equations, we obtain the following expression for  $P_n$  ( $T_k \leq n < T_{k+1}$ ).

$$P_n = P_0 \frac{(\rho \alpha_k)^n}{n!} \prod_{j=1}^k \left( \frac{\alpha_j - 1}{\alpha_j} \right)^{T_j}, \quad (6)$$

where  $P_0$  is the probability that all channels are free and obtained from equation  $\sum_{n=0}^C P_n = 1$  and can be expressed by the following expression.

$$P_0 = \left[ \sum_{k=0}^{N-1} \prod_{j=1}^k \left( \frac{\alpha_j - 1}{\alpha_j} \right)^{T_j} \sum_{n=T_k+1}^{T_{k+1}} \frac{(\rho \alpha_k)^n}{n!} \right]^{-1}. \quad (7)$$

Thus, the blocking probability of class  $N$  is equal to

$$B_N(T) = P_C, \quad B_N(T) = \prod_{i=1}^{N-1} \left( \frac{\alpha_{i-1}}{\alpha_i} \right)^{T_i} \frac{(\rho \alpha_{N-1})^{T_N}}{T_N!} P_0. \quad (8)$$

Similarly, the blocking probability of class  $k$  ( $k < N$ ) is given by

$$B_k(T) = \sum_{n=T_k+1}^C P_n, \quad (9)$$

$$B_k(T) = \sum_{j=k}^{N-1} \prod_{i=1}^j \left( \frac{\alpha_{i-1}}{\alpha_i} \right)^{T_i} \sum_{n=T_j+1}^{T_{j+1}} \frac{(\rho \alpha_j)^n}{n!} P_0.$$

From the above equation, it is evident that  $B_k(T) \geq B_{k+1}(T)$  (for  $1 \leq k < N$ ). Blocking probabilities also have the following useful properties assuming that all other system parameters are fixed. The proofs of these properties are given in Appendix A.

**Property 1.** For any given value of  $T$ ,  $B_k(\cdot)$  is a monotonically decreasing function of  $T_k$  provided that

$$\frac{\alpha_{k-1} - \alpha_k}{\alpha_{k-1}} < \left( \frac{\alpha_{N-1}}{\alpha_k} \right)^{T_N} \times \frac{1}{T_N - T_k} \quad \text{and} \quad \frac{\rho \alpha_k}{T_k + 2} < 1.$$

**Property 2.** For any given value of  $T$ ,  $B_k(\cdot)$  is a monotonically increasing function of  $T_j$  ( $j \neq k$ ).

In the rest of this section, we study the effect of arrival rates on the blocking probabilities of different classes through simulations. Figs. 2–5 show the effect of arrival rate of class 1 on blocking probabilities of different classes when  $\lambda_2=3$ ,  $\lambda_3=5$ , and  $\lambda_4=2$ , and Figs. 6–9 show the effect of arrival rate of class 1 on blocking probabilities of different classes when  $\lambda_1=6$ ,  $\lambda_3=5$ , and  $\lambda_4=2$ .

Figs. 10–13 show the effect of arrival rate of class 3 on blocking probabilities of different classes when  $\lambda_1=6$ ,  $\lambda_2=3$ , and  $\lambda_4=2$ , and Figs. 14–17 show the effect of arrival rate of class 4 on blocking probabilities of different classes when  $\lambda_1=6$ ,  $\lambda_2=3$ , and  $\lambda_3=5$ .

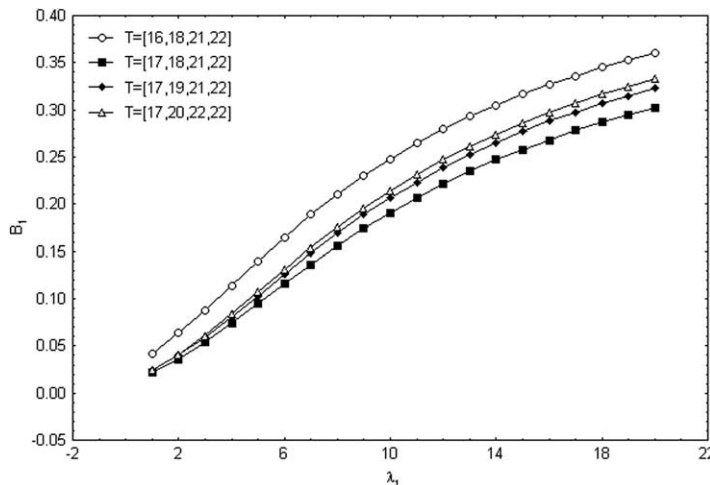


Fig. 2. The effect of  $\lambda_1$  on the blocking probability of class 1.

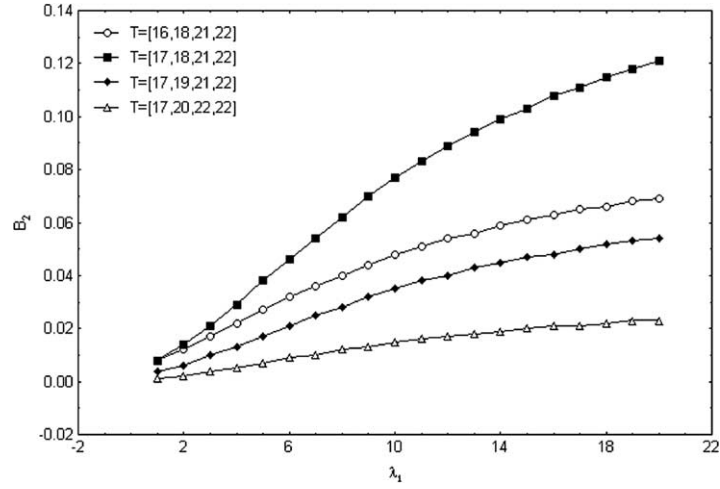


Fig. 3. The effect of  $\lambda_1$  on the blocking probability of class 2.

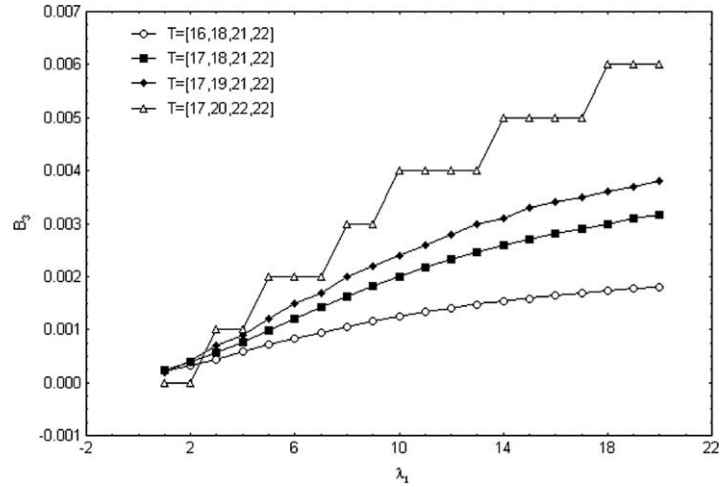


Fig. 4. The effect of  $\lambda_1$  on the blocking probability of class 3.

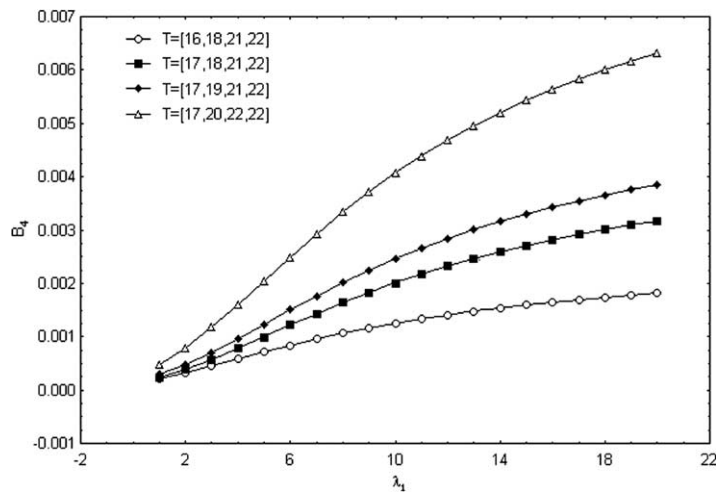


Fig. 5. The effect of  $\lambda_1$  on the blocking probability of class 4.

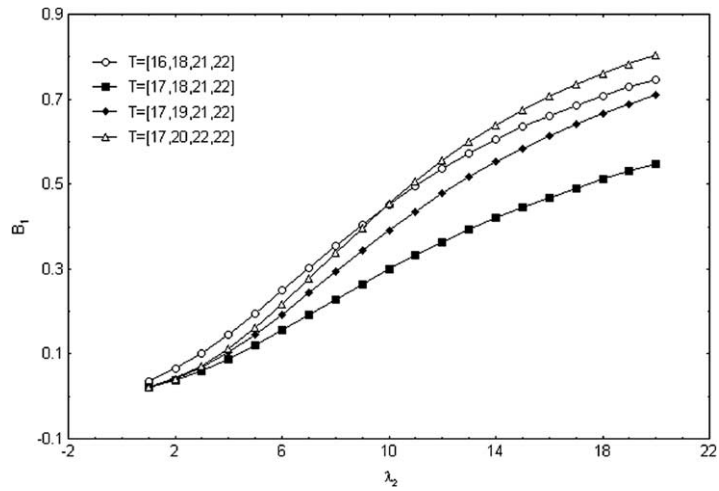


Fig. 6. The effect of  $\lambda_2$  on the blocking probability of class 1.

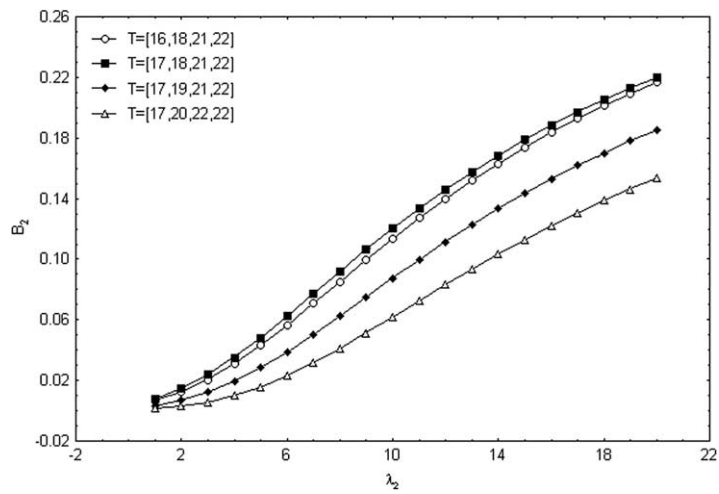


Fig. 7. The effect of  $\lambda_2$  on the blocking probability of class 2.

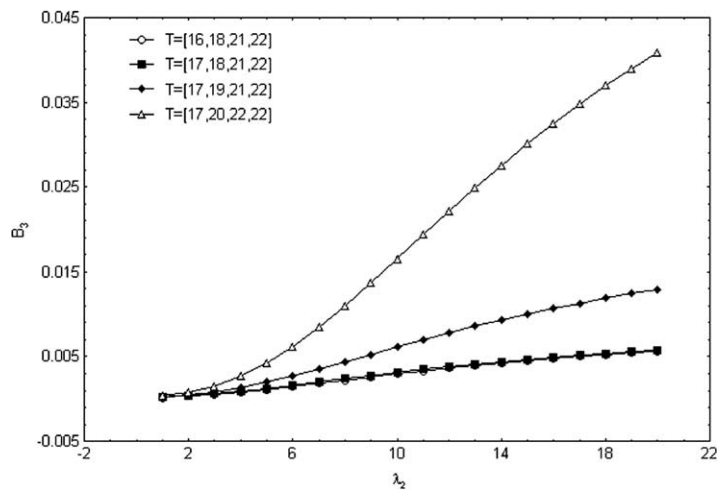


Fig. 8. The effect of  $\lambda_2$  on the blocking probability of class 3.

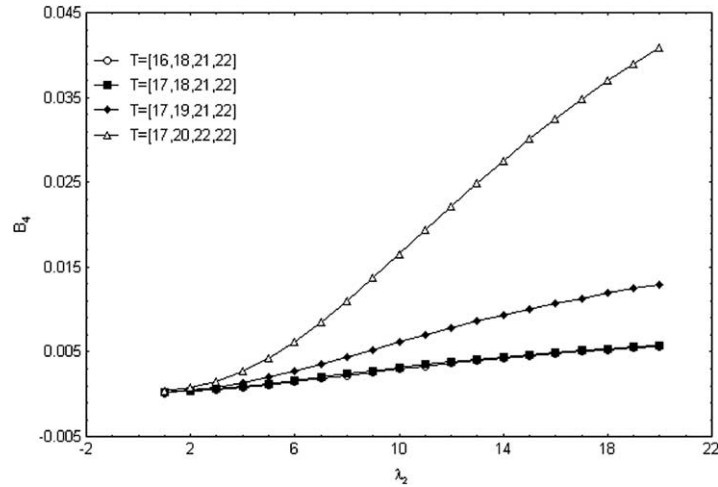


Fig. 9. The effect of  $\lambda_2$  on the blocking probability of class 4.

For all the above simulations we have considered unit channel holding time for all calls, that is  $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 1$ .

$$E_i^j = \sum_{k=i}^j e_k.$$

### 3. Optimal number of guard channels

In this section, we consider the problem of finding the optimal values of thresholds  $T_1, \dots, T_{N-1}$ , which can be described as: given  $C$  channels allocated to a cell, the objective is to find  $T^*$  that minimizes  $B_1(\cdot)$  subject to constraints  $B_k(\cdot) \leq q_k$  (for  $k=2, \dots, N$ ). Thus, we have the following nonlinear integer programming problem.

**Problem 1.** Minimize  $B_1(T)$  subject to the constraint  $B_k(T) \leq q_k$ , where  $q_k$  is the level of QoS to be satisfied for calls of class  $k$  (for  $k=2, \dots, N$ ).

We now present an algorithm called *MinBlock* (Algorithm 2) for solving problem 1. Let

The *MinBlock* algorithm uses a function called *MinBlockCheck*. Function *MinBlockCheck* ( $T, k$ ) increments  $T_j$  ( $j > k$ ) by one if the specified level of QoS for class  $j$  is satisfied and returns *true*; otherwise it returns *false*, where  $j$  ( $k+1 \leq j \leq N-1$ ) is the largest possible value which satisfies the specified level of QoS.

**Algorithm 2** Algorithm *MinBlock* for finding optimal parameters for multi-threshold guard channel scheme

- 1:  $T_0 \leftarrow -1$ ;
- 2:  $T_1 \leftarrow T_2 \leftarrow \dots \leftarrow T_N \leftarrow C$
- 3: **if**  $B_N(T) \leq q_N$  **then**
- 4:     **return**  $T$
- 5: **end if**
- 6: **for**  $k \leftarrow N$  **down to** 2 **do**
- 7:     **while**  $T_{k-1} > 0$  **and**  $B_k(T) > q_k$  **do**
- 8:         **if not** *MinBlockCheck*( $T, k+1$ ) **then**

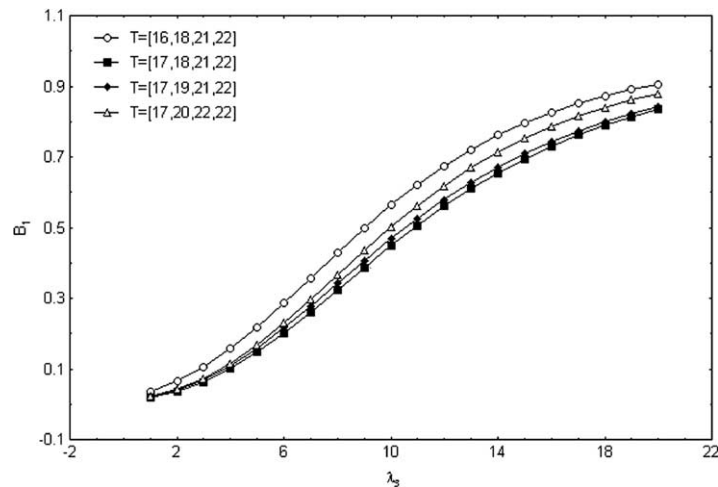


Fig. 10. The effect of  $\lambda_3$  on the blocking probability of class 1.

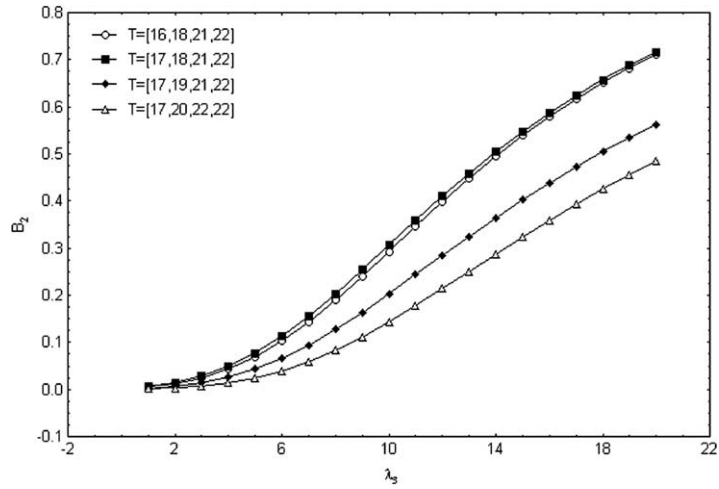


Fig. 11. The effect of  $\lambda_3$  on the blocking probability of class 2.

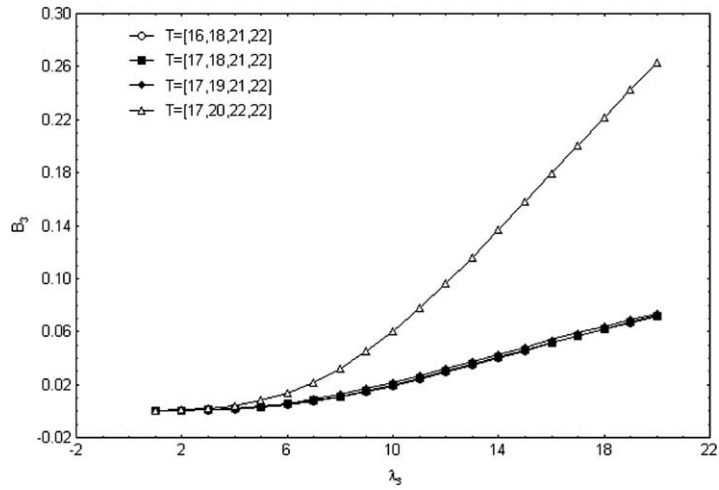


Fig. 12. The effect of  $\lambda_3$  on the blocking probability of class 3.

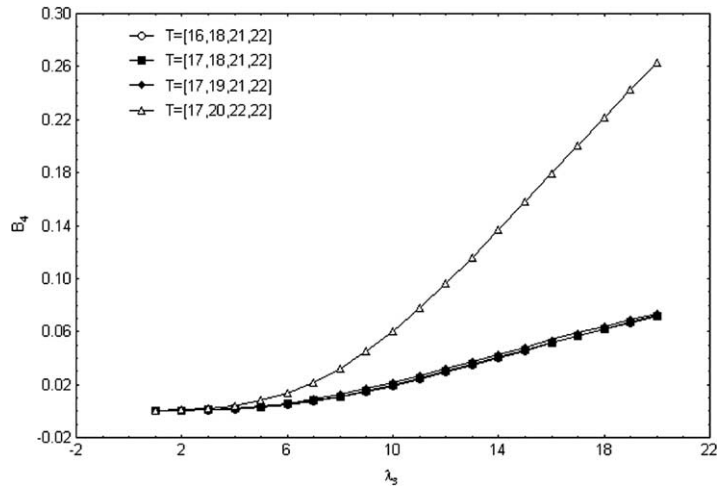


Fig. 13. The effect of  $\lambda_3$  on the blocking probability of class 4.



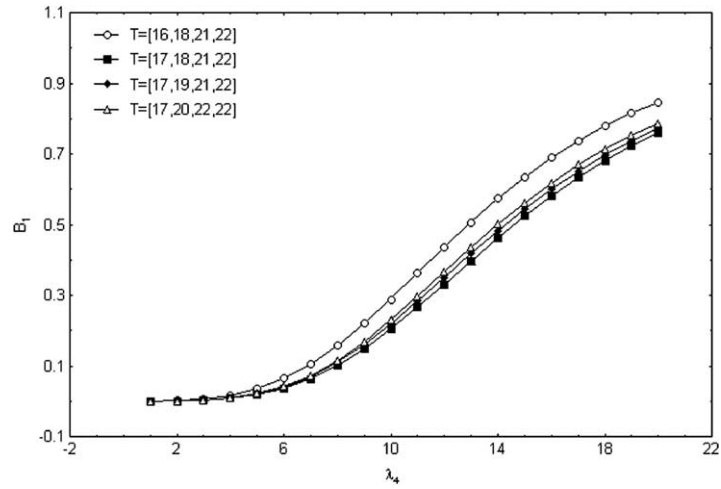


Fig. 14. The effect of  $\lambda_4$  on the blocking probability of class 1.

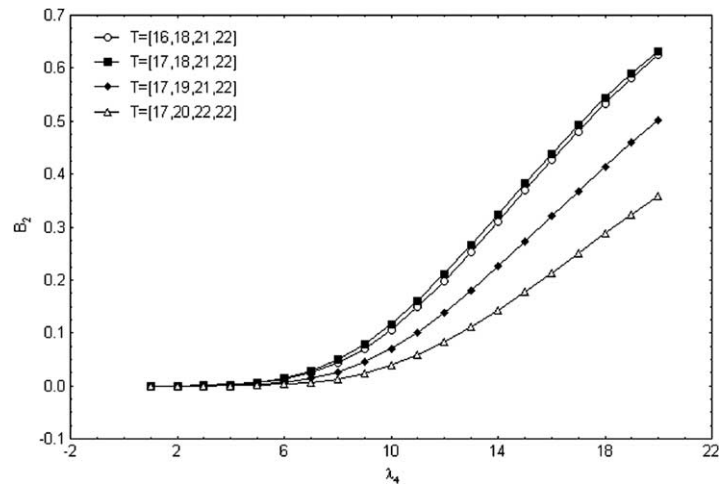


Fig. 15. The effect of  $\lambda_4$  on the blocking probability of class 2.

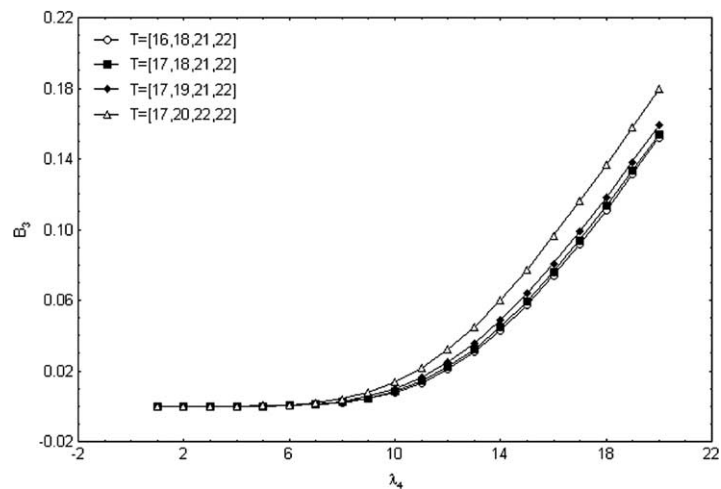


Fig. 16. The effect of  $\lambda_4$  on the blocking probability of class 3.

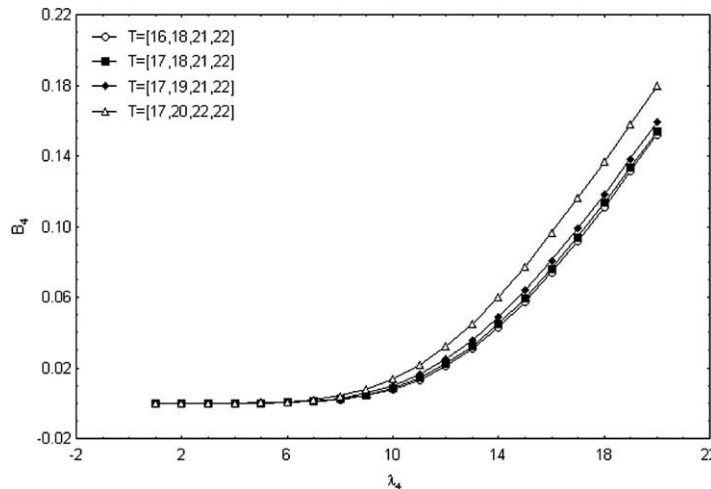


Fig. 17. The effect of  $\lambda_4$  on the blocking probability of class 4.

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9:    $T \leftarrow T - E_1^{k-1}$ 
10:  end if
11:  end while
12:  end for
13:  if at least one of constraints  $q_2, \dots, q_N$  isn't satisfied
14:  then
15:    return Number of channels is too small.
16:  end if
17:  return T
18:  function MinBlockCheck(T,k)
19:    if  $k=N$  and  $T_{k-1} < T_k$  and  $B_k(T + e_{k-1}) \leq q_k$  then
20:       $T \leftarrow T + e_{k-1}$ 
21:      return true
22:    else if  $K < N$  and not MinBlockCheck(T,k + 1) and
23:       $T_{k-1} < T_k$  and  $B_k(T + e_{k-1}) \leq q_k$  then
24:       $T \leftarrow T + e_{k-1}$ 
25:      return true
26:    end if
27:    return false
28:  end function

```

**Theorem 1.** Algorithm *MinBlock* minimizes  $B_1(\cdot)$  while satisfying constraints  $B_k(\cdot) \leq q_k$  for  $k=2, \dots, N$ .

**Proof.** Since  $B_k$  (for  $k=1, \dots, N$ ) is a decreasing function of  $T_k$  (property 1), it is sufficient to show that the *MinBlock* maximizes  $T_1$  while satisfying the specified levels of QoS. The proof is by induction on  $N$ . We shall show that if algorithm *MinBlock* minimizes  $B_1(\cdot)$  for  $N$  classes, then it still minimizes  $B_1(\cdot)$  for  $N+1$  classes and when it stops,  $T$  is the optimal solution for problem 1.

**Basis:** For  $N=2$ , function *MinBlockCheck* always return false. Hence, statements in lines 7–11 perform a linear search and find the largest value of  $T_1$  satisfying the level of QoS,  $q_2$ . Since  $B_1(\cdot)$  is a decreasing function of  $T_1$ , thus the *MinBlock* algorithm minimizes  $B_1(\cdot)$ .

**Induction step:** Consider any  $N > 2$  and assume that the induction hypothesis is true for all  $m$  ( $1 \leq m \leq N$ ). In order to prove the correctness of *MinBlock* for  $N+1$

classes, we consider finding of  $T_k$  (for  $k=1, \dots, N$ ). Statements in lines 7–11 of *MinBlock* use a sequential search for finding the largest value of  $T_k$ . Since  $B_j$  (for  $2 \leq j \leq N+1$ ) is an increasing function of  $T_k$  ( $k < j$ ), thus decreasing  $T_k$  ( $k < j$ ) also decreases  $B_j$ . This contradicts the objective of *MinBlock* which is finding the largest value of  $T_k$ . This contradiction is removed in each iteration of the while-loop in line 7, by calling function *MinBlockCheck*. Thus the while-loop in line 7 finds the largest value of  $T_j$  ( $j > k \geq 2$ ). From property 1, it is concluded that the *MinBlock* algorithm minimizes  $B_1(\cdot)$ . This completes the proof of this theorem.  $\square$

**Example 1.** In Table 1, the effectiveness of multi-threshold guard channel scheme is shown for four traffic classes ( $N=4$ ). This example assumes that each cell has 20 full duplex channels ( $C=20$ ) and the constraints are  $q_2=0.035$ ,  $q_3=0.02$  and  $q_4=0.01$ . The column  $k$  ( $2 \leq k \leq 5$ ) shows the normalized arrival rate  $\lambda_k$  (normalized to the call holding time). Columns 6 and 7 show the optimal number of guard channels and blocking probabilities for different classes, respectively. By carefully inspecting Table 1, it is evident that all constraints are satisfied.

#### 4. Minimizing the number of channels with hard constraints

In this section, we consider the problem of finding a call admission control scheme that minimizes the number of channels while satisfying constraints  $B_k(\cdot) \leq q_k$  (for all  $k=1, \dots, N$ ). Thus, we have the following nonlinear optimization problem.

**Problem 2.** Minimize  $C$  such that constraints  $B_k(T) \leq q_k$  (for  $k=1, \dots, N$ ) are satisfied.

Algorithm 3 solves problem 2 and assures the pro-specified level of QoS of calls. This algorithm, which is

Table 1  
Results of simulation for finding the optimal number of guard channels

Case	$A_0$	$A_1$	$A_2$	$A_3$	$T_1$	$T_2$	$T_3$	$T_4$	$B_1(T)$	$B_2(T)$	$B_3(T)$	$B_4(T)$
1	17	12	7	2	18	18	18	20	0.015822	0.015822	0.015822	0.001438
2	18	8	7	2	18	18	18	20	0.018911	0.018911	0.018911	0.001719
3	20	10	5	2	15	17	18	20	0.267492	0.027544	0.002858	0.000260
4	20	12	7	2	13	17	18	20	0.513851	0.031402	0.003259	0.000296
5	20	10	7	2	14	17	18	20	0.346408	0.025677	0.002665	0.000242
6	21	11	3	2	16	17	18	20	0.196272	0.030776	0.003194	0.000290
7	22	12	7	2	13	17	18	20	0.540299	0.033018	0.003426	0.000311
8	25	15	7	2	9	18	19	20	0.940582	0.034587	0.003144	0.003144

called *MinChannels*, uses function *MinChannelsCheck*. The *MinChannelsCheck*( $T, k$ ) increments  $T_j$  ( $j > k$ ) by one if the specified level of QoS,  $q_j$ , is not satisfied and returns *true*, where  $j$  ( $k < j \leq N$ ) is the largest possible value for  $j$ .

**Theorem 2.** Algorithm *MinChannels* minimizes the number of channels used by each cell and also finds threshold  $T$  in such a way that constraints  $B_k(T) \leq q_k$  (for  $k=1, \dots, N$ ) are satisfied.

**Proof.** Since  $B_k$  (for  $k=1, \dots, N$ ) is a decreasing function of  $T_k$ , it is sufficient to show that the *MinChannels* algorithm minimizes  $T_k$  while specified levels of QoS are satisfied. The proof is by induction on  $N$ . We shall show that if the algorithm minimizes  $T_N$  for  $N$  classes, then it minimizes  $T_{N+1}$  for  $N+1$  classes.

**Basis:** When we have one class, the *Minchannels* algorithm increments  $T_1$  one by one until the level of QoS is satisfied. Since the initial value of  $T_1$  is zero and incremented by one in each iteration, thus the smallest value for  $T_1$  (the optimal value) is obtained.

**Induction step:** Consider any  $N > 1$  classes and assume that the induction hypothesis is true for all  $m$  classes such that  $1 \leq m \leq N$ . In order to prove the correctness of *MinChannels* for  $N+1$  classes, we consider finding of  $T_k$  (for  $k=1, \dots, N+1$ ).

**Algorithm 3** Algorithm *MinChannels* for finding the minimum number of channels for multi-threshold guard channel scheme.

```

1:  $T_0 \leftarrow -1$ ;
2:  $T_1 \leftarrow T_2 \leftarrow \dots \leftarrow T_N \leftarrow 0$ 
3: while at least one constraint isn't satisfied do
4:   MinChannelCheck( $T, 1$ )
5: end while
6: for  $k \leftarrow N-1$  down to 1 do
7:   while  $T_k < T_{k+1}$  and all constraints for  $T+e_k$  are
   satisfied do
8:      $T \leftarrow T+e_k$ 
9:   end while
10: end for
11: return  $T$ 
1: function MinChannelCheck( $T, k$ )
2:   if  $k=N$  and  $B_N(T) > q_N$  then
3:      $T \leftarrow T+e_N$ 

```

```

4:   return true
5:   else if  $k < N$  and not MinChannelCheck( $T, k+1$ )
and  $T_k < T_{k+1}$  and  $B_k(T) > q_k$  then
6:      $T \leftarrow T+e_k$ 
7:   return true
8:   end if
9:   return false
10: end function

```

From the algorithm, it is clear that the statements in lines 3–5 use sequential search for finding the smallest value of  $T_k$  (for  $k=1, \dots, N+1$ ) until specified levels of QoS's are satisfied. Since  $B_k$  (for  $1 \leq k \leq N+1$ ) is a decreasing function of  $T_k$ , the *MinChannelsCheck* increases  $T_k$  if QoS on  $B_k$  is not satisfied. However  $B_j$  ( $1 \leq j \leq N+1$ ) increases as  $T_k$  ( $j \neq k$ ) increases. This side effect is removed in *MinChannelsCheck* by incrementing only one  $T_k$  on each of its call, where  $k$  is the largest possible value such that its level of QoS is not satisfied. Since on each call of *MinChannelsCheck* only one threshold is incremented and also the side effects are removed, thus the while-loop in lines 3–5 finds the smallest value for each threshold, including  $T_{N+1}$ , while satisfying the specified level of QoS. Hence the *Minchannels* algorithm finds the minimum number of channels ( $T_{N+1}$ ). This completes the proof of this theorem.  $\square$

**Example 2.** In Table 2, the result of *MinChannels* algorithm is given. This example assumes that we have four traffic classes with QoS's  $q_1=0.04$ ,  $q_2=0.03$ ,  $q_3=0.02$ , and  $q_4=0.01$ . The column  $k$  ( $2 \leq k \leq 5$ ) shows the arrival rate  $A_k$ , which is normalized to the call holding time. The next columns show the optimal number of guard channels and blocking probabilities for different classes, respectively.

## 5. Optimal channel assignment in multi-cells cellular networks

In this section, we extend problem 1 to multi-cells cellular networks and introduce a prioritized channel assignment algorithm. We consider a multi-cells network consisting of several clusters, where a typical cluster  $m$  contains  $N_m$  cells. Assume that a total of  $C$  full duplex channels are allocated to the whole network and hence to

Table 2  
Results of simulation for finding the minimum number of channels required to satisfy the specified level of QoS

Case	$A_0$	$A_1$	$A_2$	$A_3$	$T_1$	$T_2$	$T_3$	$T_4$	$B_1(T)$	$B_2(T)$	$B_3(T)$	$B_4(T)$
1	17	12	4	1	14	14	14	16	0.019769	0.019769	0.019769	0.001163
2	21	16	15	12	27	27	28	29	0.028000	0.028000	0.008195	0.008195
3	22	17	9	1	17	17	17	19	0.017661	0.017661	0.017661	0.000883
4	23	18	15	12	29	29	30	31	0.026658	0.026658	0.007439	0.007439
5	26	21	13	5	28	28	28	30	0.019735	0.019735	0.019735	0.002819
6	28	23	15	12	34	34	35	36	0.023509	0.023509	0.005877	0.005877
7	30	25	17	5	32	32	32	34	0.016455	0.016455	0.016455	0.002110
8	36	31	23	15	43	43	44	45	0.024401	0.024401	0.006100	0.006100

each cluster. The proposed prioritized channel assignment algorithm divides the set of all channels into  $N_m$  disjoint channel sets, where each channel set is allocated to one cell in the cluster. Then, the channel set of each cell is divided into  $N$  subsets by employing  $N - 1$  thresholds. By applying this algorithm to each cluster, the prioritized channel assignment is obtained for the whole network. Let

$$A = \sum_{i=1}^{N_m} \lambda_1^i$$

be the total arrival rate of class 1 over all cells in cluster  $m$  and  $\lambda_1^i$  is the arrival rate of class 1 calls in cell  $i$  of cluster  $m$ . Define the overall blocking probability of calls of class 1 by

$$B = \sum_{i=1}^{N_m} \frac{\lambda_1^i}{A_n} B_1^i(T^i), \tag{10}$$

where  $B_1^i(T^i)$  is the blocking probability of calls of class 1 in cell  $i$  when  $C^i$  channels are allocated to that cell and  $T^i$  is the set of thresholds for that cell. The objective is to find the optimal value for  $T^i$  ( $i = 1, 2, \dots, N_m$ ), which minimizes the overall blocking probability of class 1 subject to the constraint on other classes. This problem is formulated by the following non-linear optimization problem.

**Problem 3.** Minimize the overall blocking probability of class 1, subject to the following constraints.

$$B_k^i(T^i) \leq q_k \quad \text{for } k = 2, \dots, N, \tag{11}$$

$$\sum_{i=1}^{N_m} C^i = C, \tag{12}$$

for all cells  $i = 1, 2, \dots, N_m$  in cluster  $m$ .

In what follows, we propose a greedy algorithm for solving problem 3. This algorithm can be described as follows. Initially for each cell  $i$ , the smallest number of channels required to satisfy the given QoS is found. To do this, we use algorithm *MinChannels* with the constraint  $q_1 = 1 - \epsilon$ , where  $\epsilon \ll 1$  is a small positive value. Then the remaining channels, if any, are allocated to cells one by one. Let  $\gamma_i$  denotes the amount of decrement in  $B_1^i$  brought by allocation of an additional channel to cell  $i$ . Note that the additional channel can be used in any  $N$  subsets. In order to

find the usage of the additional channel, the *MinBlock* algorithm is used. The amount of decrement in  $B_1^i$  are computed for all cell  $i$  (for  $i = 1, 2, \dots, N_m$ ) according to the following equation.

$$\gamma_i = \frac{\lambda_1^i}{A} [B_1^i(T^i) - B_1^i(T^i + e_N)],$$

where  $T^i$  (for  $i = 1, \dots, N$ ) are threshold sets obtained by the *MinBlock* algorithm. Note that  $\gamma_i$  is always positive. Then a cell with the largest decrement in  $B_1(\cdot)$  is found among all cells in the cluster and an additional channel is assigned to it. This procedure is repeated until all available channels  $C$  in the cluster are used. Algorithm 4 summarizes this procedure.

**Theorem 3.** Algorithm 4 finds the optimal solution for problem 3.

**Algorithm 4** Algorithm *MinBlock* for multi-cells channel assignment

Use *MinChannels* for all cells  $1 \leq i \leq N_m$  with constraint  $q_1 = 1 - \epsilon$ , where  $\epsilon$  is a small positive value.

$$S \leftarrow C - \sum_{i=1}^{N_m} C^i$$

```

if  $S = 0$  then
    terminate algorithm,  $T$  is optimal.
end if
if  $S < 0$  then
    terminate algorithm,  $C$  channels cannot satisfy the
    specified QoS.
end if
for  $i \leftarrow 1$  to  $N_m$  do
    Use MinBlock for cell  $i$  with  $C^i$  and  $C^i + 1$  channels.

     $\gamma_i \leftarrow \frac{\lambda_1^i}{A} [B_1^i(T^i) - B_1^i(T^i + e_N)].$ 
end for
for  $i \leftarrow 1$  to  $S$  do
     $j \leftarrow \operatorname{argmax}_i \gamma_i.$ 
     $C^j \leftarrow C^j + 1.$ 
    
```

$$\gamma_j \leftarrow \frac{\lambda_1^j}{A} [B_1^j(T^j) - B_1^i(T^j + e_N)].$$

end for

{ $T^i | i = 1, 2, \dots, N_m$ } is the optimal solution.

**Proof.** The initial assignment is an undominated solution, in the sense that it uses the minimum number of channels to satisfy the constraints (11), which results in the maximum value of  $B$  subject to the constraints  $B_k^i(T^i) \leq q_k$  (for  $k = 2, \dots, N$ ). Then the algorithm assigns channels one by one to cells which results in the largest decrement in blocking probability of new calls. This strategy results in the optimal solution. Let  $j_i$  be the index of the cell with the largest decrement in  $B$  at stage  $i$  (for  $i = 1, 2, \dots, S$ ). Assume that there is another strategy which is optimal and at stage  $i$  chooses cell  $k_i \neq j_i$ . Thus, there is a  $\delta_i \geq 0$  for which we have  $\gamma_{k_i} = \gamma_{j_i} + \delta_i$ . Then interchanging cell  $j_i$  with cell  $k_i$  results in an assignment with the following overall blocking probability.

$$B_N^{k_i} = \sum_{l=1}^{N_m} \frac{\lambda_1^l}{A} B_1^l(T^l).$$

By subtracting  $B^{j_i}$  from  $B^{k_i}$ , we obtain

$$B^{k_i} - B^{j_i} = \delta_i.$$

Repeating this procedure for  $S$  stages, we obtain

$$\sum_{i=1}^S [B^{k_i} - B^{j_i}] = \sum_{i=1}^S \delta_i,$$

which is positive. Thus, no index other than the index with the largest value of  $\gamma_i$  would result in the optimal solution. Hence, the proposed cell selection mechanism minimizes the value of  $B$  subject to the constraints (11) and results in the optimal solution.  $\square$

**Example 3.** Consider a cellular system with 4-cells clusters. Assume that a total of 80 full duplex channels are available in this system. The level of QoS for classes 1–4 are 0.05, 0.035, 0.02, and 0.01, respectively. The call arrival rates, which are normalized to the call holding time, are given in Table 3. The result of algorithm 3 is given in Table 4.

**Remark 1.** Extension of problem 2 to multi-cells cellular networks has no engineering profit, because the number of channels allocated to the network is fixed and minimizing the number of channels wastes the system resources.

Table 3  
The traffic parameters of cellular network

Cell	$A_0$	$A_1$	$A_2$	$A_3$
1	14	9	5	2
2	14	10	5	3
3	30	28	18	5
4	18	12	9	5

Table 4  
The result of channel assignment for multi-cells system

Cell	$T_1$	$T_2$	$T_3$	$T_4$	$B_1(T)$	$B_2(T)$	$B_3(T)$	$B_4(T)$
1	12	13	13	15	0.180902	0.025208	0.025208	0.002966
2	14	15	15	17	0.131336	0.023735	0.023735	0.00356
3	21	22	22	24	0.119553	0.024871	0.024871	0.004288
4	22	22	22	24	0.016902	0.016902	0.016902	0.002914

### 6. Conclusions

In this paper, a call admission problem in cellular networks that support several classes of calls was studied. First, we introduced a multi-threshold guard channel policy and studied its limiting behavior under the stationary traffic. Then we gave an algorithm for finding the optimal number of guard channels that minimizes the blocking probability of calls with the lowest level of QoS subject to constraints on blocking probabilities of other calls. Second, we gave an algorithm to find the minimum number of channels subject to constraints on blocking probabilities of calls. Finally, we proposed a prioritized channel assignment algorithm for multi-cells cellular networks to minimize the blocking probability of calls with lowest level of QoS subject to constraints on blocking probabilities of other calls. Computer simulations were conducted to verify the theoretical studies presented in this paper.

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### Appendix A

In this appendix, we present proofs for properties 1 and 2. Before we present these proofs, we give some notations and state a property

$$\phi(T) = \frac{(\rho \alpha_{N-1})^{T_N}}{T_N!} \prod_{i=1}^{N-1} \left( \frac{\alpha_{i-1}}{\alpha_i} \right)^{T_i}, \tag{A.1}$$

$$G_k(T) = \prod_{i=1}^{k-1} \left( \frac{\alpha_{i-1}}{\alpha_i} \right)^{T_i} \sum_{n=T_{k-1}+1}^{T_k} \frac{(\rho \alpha_{k-1})^n}{n!}, \tag{A.2}$$

$$G(T) = \sum_{j=1}^N G_j(T), \tag{A.3}$$

$$M_k^j = \sum_{i=k+1}^j G_i(T). \tag{A.4}$$

$G(\cdot)$  have the following property, assuming that all other system parameters are fixed.

Property 3. For any given value of  $T$ , we have

$$G(T + e_j) = \sum_{k=1}^j G_k(T) + \frac{\alpha_{j-1}}{\alpha_j} \sum_{k=j+1}^N G_k(T),$$

for  $j=1,2,\dots,N-1$ .

Proof. Since  $G_1(\cdot), \dots, G_{j-1}(\cdot)$  are independent of  $T_j$ , we have

$$G(T + e_j) = \sum_{k=1}^{j-1} G_k(T) + G_j(T + e_j) + \sum_{k=j+1}^N G_k(T + e_j), \tag{A.5}$$

$$G(T + e_j) = \sum_{k=1}^j G_k(T) + \frac{(\rho\alpha_{j-1})^{T_j+1}}{(T_j + 1)!} \times \prod_{i=1}^{j-1} \left(\frac{\alpha_{i-1}}{\alpha_i}\right)^{T_i} + \sum_{k=j+1}^N G_k(T + e_j), \tag{A.6}$$

$$G(T + e_j) = \sum_{k=1}^j G_k(T) + \left(\frac{\alpha_{j-1}}{\alpha_j}\right) \frac{(\rho\alpha_j)^{T_j+1}}{(T_j + 1)!} \times \prod_{i=1}^j \left(\frac{\alpha_{i-1}}{\alpha_i}\right)^{T_i} + \sum_{k=j+1}^N G_k(T + e_j), \tag{A.7}$$

Using (A.2), the last term of the above equation can be written as

$$\sum_{k=j+1}^N G_k(T + e_j) = \left(\frac{\alpha_{j-1}}{\alpha_j}\right) \left[ \sum_{k=j+1}^N G_k(T) - \frac{(\rho\alpha_j)^{T_j+1}}{(T_j + 1)!} \prod_{i=1}^j \left(\frac{\alpha_{i-1}}{\alpha_i}\right)^{T_i} \right]. \tag{A.8}$$

Substituting the above equation in Eq. (A.7), we obtain

$$G(T + e_j) = \sum_{k=1}^j G_k(T) + \frac{\alpha_{j-1}}{\alpha_j} \sum_{k=j+1}^N G_k(T). \tag{A.9}$$

□

### A.1. Proof of Property 1

Let

$$Z_k = \prod_{i=1}^k \left(\frac{\alpha_{i-1}}{\alpha_i}\right)^{T_i} \frac{(\rho\alpha_k)^{T_k+1}}{T_{k+1}!}.$$

Since

$$\frac{\alpha_{k-1} - \alpha_k}{\alpha_{k-1}} < \left(\frac{\alpha_{N-1}}{\alpha_k}\right)^{T_N} \times \frac{1}{T_N - T_k} \text{ and } \frac{\rho\alpha_k}{T_k + 2} < 1,$$

we have

$$\frac{\alpha_{k-1} - \alpha_k}{\alpha_{k-1}} < \left(\frac{\alpha_{N-1}}{\alpha_k}\right)^{T_N} \times \frac{1}{T_N - T_k} \tag{A.10}$$

$$\frac{\alpha_{k-1} - \alpha_k}{\alpha_{k-1}} < \frac{\prod_{i=k+1}^{N-1} \left(\frac{\alpha_i}{\alpha_{i-1}}\right)^{T_i}}{1 + \dots + 1} \tag{A.11}$$

$$\frac{\alpha_{k-1} - \alpha_k}{\alpha_{k-1}} < \frac{\prod_{i=k+1}^{N-1} \left(\frac{\alpha_i}{\alpha_{i-1}}\right)^{T_i}}{1 + \dots + \left(\frac{\rho\alpha_k}{T_{k+2}}\right)^{T_N - T_k - 1}} \tag{A.12}$$

$$\frac{\alpha_{k-1} - \alpha_k}{\alpha_{k-1}} < \frac{\prod_{i=k+1}^{N-1} \left(\frac{\alpha_i}{\alpha_{i-1}}\right)^{T_i}}{1 + \dots + \frac{(\rho\alpha_k)^{T_N - T_k - 1}}{(T_{k+2}) \times \dots \times T_N}} \tag{A.13}$$

$$\frac{\alpha_{k-1} - \alpha_k}{\alpha_{k-1}} = \frac{1}{\prod_{i=k+1}^{N-1} \left(\frac{\alpha_{i-1}}{\alpha_i}\right)^{T_i} \left[1 + \dots + \frac{(\rho\alpha_k)^{T_N - T_k - 1}}{(T_{k+2}) \times \dots \times T_N}\right]}. \tag{A.14}$$

Multiplying and dividing the above inequality by  $Z_k$ , we obtain

$$\frac{\alpha_{k-1} - \alpha_k}{\alpha_{k-1}} < \frac{Z_k}{Z_k} \times \frac{1}{\prod_{i=k+1}^{N-1} \left(\frac{\alpha_{i-1}}{\alpha_i}\right)^{T_i} \left[1 + \dots + \frac{(\rho\alpha_k)^{T_N - T_k - 1}}{(T_{k+2}) \times \dots \times T_N}\right]} \tag{A.15}$$

$$\frac{\alpha_{k-1} - \alpha_k}{\alpha_{k-1}} = \frac{Z_k}{\prod_{i=1}^{N-1} \left(\frac{\alpha_{i-1}}{\alpha_i}\right)^{T_i} \sum_{n=T_k+1}^{T_N} \frac{(\rho\alpha_k)^n}{n!}} \tag{A.16}$$

$$\frac{\alpha_{k-1} - \alpha_k}{\alpha_{k-1}} < \frac{Z_k}{\prod_{i=1}^{N-1} \left(\frac{\alpha_{i-1}}{\alpha_i}\right)^{T_i} \sum_{n=k}^{N-1} \sum_{n=T_m+1}^{T_{m+1}} \frac{(\rho\alpha_m)^n}{n!}} \tag{A.17}$$

$$\frac{\alpha_{k-1} - \alpha_k}{\alpha_{k-1}} < \frac{Z_k}{\sum_{m=k}^{N-1} \prod_{i=1}^m \left(\frac{\alpha_{i-1}}{\alpha_i}\right)^{T_i} \sum_{n=T_m+1}^{T_{m+1}} \frac{(\rho\alpha_m)^n}{n!}} \tag{A.18}$$

$$\frac{\alpha_{k-1} - \alpha_k}{\alpha_{k-1}} = \frac{Z_k}{M_k^N}. \tag{A.19}$$

From the above inequality, we obtain

$$\left[\frac{\alpha_{k-1} - \alpha_k}{\alpha_{k-1}}\right] M_k^N < Z_k < \frac{\alpha_{k-1}}{\alpha_k} Z_k. \tag{A.20}$$

Now computing  $B_k(T + e_k) - B_k(T)$  and using the above inequality, we obtain

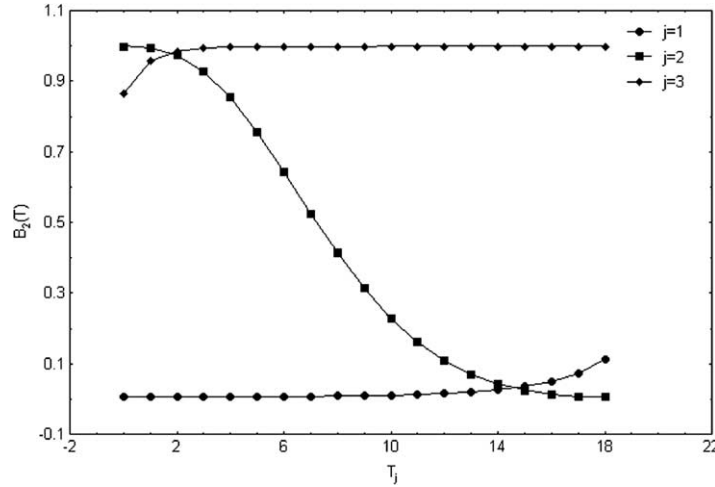


Fig. A.1. The effect of  $T$  on the blocking probabilities.

$$B_k(T + e_k) - B_k(T) = \frac{(M_k^N - Z_k) \frac{\alpha_{k-1}}{\alpha_k}}{\left(M_0^{k-1} + \frac{\alpha_{k-1}}{\alpha_k} M_k^N\right)} - \frac{M_k^N}{M_0^N} \quad (\text{A.21})$$

$$B_k(T) = \frac{M_k^N}{M_0^N} \quad (\text{A.29})$$

By computing  $B_k(T + e_j) - B_k(T)$ , we obtain

$$B_k(T + e_k) - B_k(T) < \frac{\left(\frac{\alpha_{k-1}}{\alpha_k} - 1\right) M_k^N - \frac{\alpha_{k-1}}{\alpha_k} Z_k}{M_0^N} \quad (\text{A.22})$$

$$B_k(T + e_j) - B_k(T) = \frac{M_0^{k-1} M_j^N \left(\frac{\alpha_{j-1}}{\alpha_j} - 1\right)}{\left(M_0^{j-1} + \frac{\alpha_{j-1}}{\alpha_j} M_j^N\right) M_0^N} \quad (\text{A.30})$$

$$B_k(T + e_k) - B_k(T) < 0, \quad (\text{A.23})$$

$$B_k(T + e_j) - B_k(T) > 0, \quad (\text{A.31})$$

which completes the proof of this property.  $\square$

which completes the proof of this part.

The above properties are shown graphically by Fig. A.1.  $\square$

### A.2. Proof of Property 2

The proof of property 2 has two parts.

*Proof for  $j < k$ :* From Eqs. (9) and (A.4) and property 3, we have

$$B_k(T + e_j) = \frac{\frac{\alpha_{j-1}}{\alpha_j} M_k^N}{M_0^{j-1} + \frac{\alpha_{j-1}}{\alpha_j} M_j^N} \quad (\text{A.24})$$

$$B_k(T) = \frac{M_k^N}{M_0^N} \quad (\text{A.25})$$

By computing  $B_k(T + e_j) - B_k(T)$ , we obtain

$$B_k(T + e_j) - B_k(T) = \frac{M_0^{j-1} M_k^N \left(\frac{\alpha_{j-1}}{\alpha_j} - 1\right)}{\left(M_0^{j-1} + \frac{\alpha_{j-1}}{\alpha_j} M_j^N\right) M_0^N} \quad (\text{A.26})$$

$$B_k(T + e_j) - B_k(T) > 0, \quad (\text{A.27})$$

which completes the proof of this part.

*Proof for  $j > k$ :* From Eqs. (9) and (A.4) and Property 3, we have

$$B_k(T + e_j) = \frac{M_k^{j-1} + \frac{\alpha_{j-1}}{\alpha_j} M_j^N}{M_0^{j-1} + \frac{\alpha_{j-1}}{\alpha_j} M_j^N}, \quad (\text{A.28})$$

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