Stochastic Optimization Using Continuous Action-Set Learning Automata

H. Beigy* and M.R. Meybodi1

In this paper, an adaptive random search method, based on continuous action-set learning automata, is studied for solving stochastic optimization problems in which only the noise-corrupted value of a function at any chosen point in the parameter space is available. First, a new continuous action-set learning automaton is introduced and its convergence properties are studied. Then, applications of this new continuous action-set learning automata to the minimization of a penalized Shubert function and pattern classification are presented.

INTRODUCTION

Optimization with noisy corrupted measurements is a common problem in many areas of engineering. Consider a system with measurements of \( g(x, \alpha) \), where \( \alpha \) is the parameter and \( x \) is the observation. The parameter optimization problem is defined so as to determine the optimal parameter, \( \alpha^* \), such that the performance function, \( M(\alpha) = E[g(x, \alpha)] \), is optimized. Many efficient methods, like the steepest descent method and Newton's method, are available when gradient, \( \nabla M \), is explicitly available. Usually, due to the lack of sufficient information concerning the structure of function \( M \) or because of mathematical intractability, function \( M \) to be optimized is not explicitly known and only the noise-corrupted value of function \( M(\alpha) \) at any chosen point, \( \alpha \), can be observed. Two important classes of algorithms are available for solving the optimization problem when only the noise-corrupted observations are available: Stochastic approximation based algorithms [1] and learning automata based algorithms [2,3].

Stochastic approximation algorithms are iterative algorithms in which the gradient of function, \( M \), is approximated by a finite difference method and using the function evaluations obtained at points, which are chosen close to each other [1]. Learning automata are adaptive decision making devices that operate in unknown random environments and progressively improve their performance via a learning process.

Learning automata are very useful for optimization of multi-modal functions when the function is unknown and only noise-corrupted evaluations are available. In these algorithms, a probability density function, which is defined over the parameter space, is used for selecting the next point. The reinforcement signal and the learning algorithm are used by learning automata for updating the probability density function at each stage. It is required that this probability density function converge to some probability density function where the optimal parameter, \( \alpha^* \), is chosen with probability as being as close as possible to unity. The distinguishing feature of the learning is that the probability distribution of \( g(x, \alpha) \) is unknown.

Methods based on stochastic approximation algorithms and learning automata represent two distinct approaches to learning problems. Though both approaches involve iterative procedures, updating at every stage is done in the parameter space in the first method, which may result in a local optimum, and in the probability space in the second method. Learning automata methods have two distinct advantages over the stochastic approximation algorithms. The first advantage is that the action space need not be a metric space because, as in stochastic approximation algorithms, the new value of the parameter is to be chosen close to the previous value. The second advantage is that the methods based on learning automata lead to global optimization, because, at every stage any element of the action-set can be chosen.

In this paper, an adaptive random search method for finding the global minimum of an unknown function is studied. In the first part of the paper, a new Continuous Action-set Learning Automaton (CALA) is introduced and its convergence behavior is stud-