Banyan Heap Machine

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Abstract

A priority queue is a device which stores a set of elements and their associated priorities and provides a set of operations on these elements, called priority queue operations. The standard operations on priority queue are XMAX *and* INSERT. XMAX *operation retrieves and deletes the element with the highest priority and* INSERT *operation inserts an element and its associated priority into the priority queue. A number of multiprocessor designs for maintaining priority queue have been proposed in the literatures. These designs can be classified into two main groups: 1) designs with many processors each having an small amount of memory, and 2) designs with small number of processors each having a large amount of memory. In this paper a new designs, belonging to the second group, for performing a group of priority queue operations on a set of element are presented. Processors in this design, called banyan heap machine are connected together to form a linear chain. The algorithms for Banyan Heap machine are the generalization of binary heap algorithms to a more general acyclic graph called banyan. This design, unlike existing designs, requires fewer processors to meet the same capacity requirement, and also, processors do not have geometrically varying memory sizes. This results in a completely homogeneous system. The key advantage of banyan heap machine is in its ability to retrieve elements at different percentile levels.*

Index Terms: Priority Queue, Banyan Heap, Parallel Algorithm, Parallel Computer

1 Introduction

Special classes of computational tasks have led to the development and realization of special purpose systems that most efficiently perform the given tasks. Special purpose machines for variety of computational tasks such as : signal and image processing, matrix operations, graph algorithms, database operations have been proposed in the literatures [4,8,9,10]. In this report we study the implementation of a priority queue data structure in hardware.

Priority queue is a very important data structure which has found applications in varieties of situations [17-19,22]. This data structure is a set of elements each of which has an associated number, its priority. For each element x, $p(x)$, the priority of x is a number from some linearly ordered set. Standard operations on a priority queue are *INSERT*, which inserts an element and its associated priority into the priority queue, and *XMAX*, which deletes the element with the highest priority from the queue. Let P denote the set of all element-priority pairs. Define

$$P(s) = \{(x, p(x)) | p(x) = s \text{ and } (x, p(x)) \in P\};$$

The effect of priority queue operations are as follows:

INSERT$(x, p(x))$:

$P \leftarrow P \bigcup \{(x, p(x))\}$.
Response is null.

XMAX:

$P \leftarrow P - P(p_{max})$ where $P(p_{max})$
is the pair with the highest priority.
Response is $P(p_{max})$.

A priority queue machine receives a stream of operations *(INSERT,XMAX)*, execute them in a piplined manner, and, in the case of *XMAX* operation, reports the element with the highest priority via the *I/O* port. The *response time* for an operation is the time elapsed between the initiation and completion of an operation, and the *pipeline interval* of an operation is the minimum time needed before the initiation of the next operation. The *period* of the machine is the maximum of all operation pipeline intervals.

Hardware realization of data structures have been investigated by several researchers. Leiserson [2] proposed a machine to implement priority queue operations and Bentley and Kung [1] have given an implementation of dictionary operations on a tree in which the data elements are stored in the leaves of the tree. Using X-trees, Ottmann et al. [3] designed a more efficient implementation of dictionary operations at the expense of additional wires. Atallah and Kosaraju [7] and Somani and Agarwal [5,6] have shown that dictionary operations can be implemented on a tree which does not use any links other than the binary tree links. Schmeck and Schroders [11], and Dehne and Santoro [20] have given an implementation of dictionary operations on mesh-connected array. Recently, J. H. Chang, O. H. Ibarra, M. J. Chung and K. Rao [24] have proposed systolic tree architectures for data structures

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such as stacks, queues, dequeues, priority queues, and dictionary machines. Cray and Thompson [14], Fisher [12], and Tanaka, Nozaka, and Masuyama [15] have shown that a dictionary machine can be constructed using search trees implemented on a linear array of processors. Other related designs are reported in [24-30].

In this paper, a new design, called banyan heap machine, for performing a group of priority queue operations on a set of elements is proposed. The algorithms for this machine are the generalization of heap algorithms to a more general acyclic graph called banyan. This design requires fewer processors than the existing designs [1- 7,11,20] in order to meet the same capacity requirement and unlike some of the existing designs [14,15], processors do not have geometrically varying memory sizes, resulting in a completely homogeneous system. The key advantage of banyan heap machine is in its ability to retrieve elements at different percentile levels. The rest of this paper is organized as follows. Section 2 defines banyan graphs and banyan heaps. Section 3 discuss Banyan Heap machine. Algorithms for Banyan Heap are given in section 4. In section 5 analytical formulas for percentile level of a retrieved element. The last section is the conclusion.

2 Banyan graphs and banyan heaps

A banyan graph is a Hasse diagram [21] of a partial ordering in which there is only one path from any base to any apex. A base is defined as any vertex with no arcs incident out of it and an apex is defined as any vertex with no arcs incident into it. A vertex that is neither an apex nor a base vertex is called an intermediate vertex.

An L-level banyan is a banyan in which the path from base to apex(or apex to base) is of length L. Therefore, in an L-level banyan, there are $L+1$ levels of nodes and L levels of edges. By convention, apexes are considered to be at level 0 and bases at level L. In a banyan graph, the outdegree and the indegree of a node are called spread and fanout of that node. If there is an edge between two nodes, x at level i and y at level $i+1$, then we say x is the parent of y, and y is the child of x.

Definition 1 *A banyan is called a uniform banyan if all the nodes within the same level have identical spread and fanout.*

In a uniform banyan , the fanout and spread values may be characterized by L component vectors, $F = (f_0, f_1, ..., f_{L-1})$ and $S = (s_1, s_2, ..., s_L)$, the fanout vector and spread vector, respectively, where s_i and f_i denotes the spread and fanout of a node at level i.

Definition 2 *If $s_{i+1} = f_i$, $(0 \leq i \leq L-1)$, that is $F = S$, then the banyan is called rectangular. If $s_{i+1} \neq s_i$ for some i, then the banyan is non-rectangular.*

Definition 3 *A banyan is said to be regular if $s_i = s$, $(1 \leq i \leq L)$, and $f_i = f$, $(0 \leq i \leq L-1)$, for some constant s and f. Otherwise it is said to be irregular.*

Definition 4 *A banyan is an SW-banyan if it has the following two additional properties: a) Two nodes at an intermediate level i, have either no or all common parents at level $i-1$. b) two nodes at intermediate level i have either no or all common children at level $i+1$.*

Definition 5 *An SW-banyan is said to be rectangular if it is regular and $s_i = d$, $(1 \leq i \leq L)$, and $f_i = d$, $(0 \leq i \leq L-1)$, for some constant d.*

Now we define banyan heap.

Definition 6 *An L-level banyan heap is an L-level banyan such that the priority of the element at each node is equal or greater than the priorities of the elements at each of its children.*

3 Banyan Heap Machine

Banyan heap machine is a linear array of $\log M + 1$ processors, one for each level of the heap. In this report, we study the implementation of MxM rectangular SW-banyan heap with $d = 2$ in an array of $\log M + 1$ processors where M is the number of apexes. The restriction to an MxM rectangular banyan is in the interest of simplicity of presentation.

In such banyans the number of levels is $\log M + 1$, each of which assigned to one processor with the apexes assigned to processor p_1. Figure 1 shows an example of a 4-level rectangular banyan heap and its mapping into the linear array of processors.

Each node in the banyan heap has six fields: DATA, PRIORITY, LCHILD, RCHILD, LEMPTYNODES, and REMPTYNODES. For a node, DATA field holds a element and the PRIORITY field holds the priority associated with that element, LCHILD and RCHILD holds respectively pointers to the left child and right child of that node, and LEMPTYNODES and REMPTYNODES hold the number of null nodes (nodes with no information) in the left and right subtrees of that nodes. In addition to the above six fields, each apex has another field called NEXT. This field is used to link apexes together. Initially, the DATA fields of all the nodes are set to null and the priority fields of all the nodes are set to -1. To initialize The LEMPTYNODES and REMPTYNODES fields, we first partition the heap into M disjoint binary trees. The partitioning process starts with the leftmost apex and continues in increasing order of the apex numbers. The leftmost apex is numbered 1. Partition i is the set of all nodes which are reachable from apex i by moving down the heap and are not part of partition $i-1$. The root of the binary tree in partition i is apex i. Once the partitions are determined, we initialize the REMPTYNODES and LEMPTYNODES fields of every node at level i in a given partition to $2^{\log M+1-i}-1$. Level of the root of a partition is defined to be 1. The depth of a partition is the maximum level of any node in that partition. Set of partitions and the initial settings of LEMPTYNODES and REMPTYNODES fields for an 8x8 SW-banyan is given in figure 2. REMPTYNODES and LEMPTYNODES fields are updated as data elements are inserted into and deleted from the machine.

Information about the number of empty nodes is used by the *INSERT* operation to decide which path in the heap should be followed during the insertion process. Lack of such information may lead to an overflow situation in the last processor. This happens if the *INSERT* operation moves along a path in which all the nodes are non-empty.

Definition 7 *An L-level partitioned banyan heap is an L-level banyan such that each partition of the banyan (as defined above) is a binary heap.*

Definition 8 *A partitioned L-level banyan heap is said to be full up to apex d if all the nodes in partitions j, ($j < d$), are non-null and the nodes in the remaining partitions are null.*

Definition 9 *A node in a banyan heap is said to be reachable by partition from apex i if its parent is reachable by partition from apex i. Node l at level j+1 is reachable by partition from node k at level j if node l either has non-zero REMPTYNODES and it is the right child of node l,or has non-zero LEMPTYNODES and is the left child of node l. An apex is reachable by partition from itself.*

Remark 1 The null nodes which are reachable by partition from a given apex will be filled up by insertions initiated at that apex unless the reachablity of the nodes will change by a later deletion operation initiated at some other apexes. Reachablity does not imply reachablity by partition.

Each processor in the array is equipped with send and receive instructions. They are used for communication between neighboring processors. Send(<processor>,'<instruction>') sends instruction <instruction>
to processor <processor> for execution. The execution of receive(<processor>,(information)) causes the information specified by the second argument be obtained from processor <processor> and forwarded to the requesting processor (the processor executing the receive instruction). Receive instruction is of blocking type, that is , it is not complete until a message is received from the specified processor.

In the next section we describe the implementation of priority queue operations for partitioned banyan heap. The implementation of priority queue operations for banyan heap is reported in [30].

4 Algorithms for Partitioned Banyan Heap

Insertion into a banyan heap is performed by operation *INSERT*. This operation, executed by processor p_1, first finds the leftmost partition which has at least one empty node. This can be done by checking the REMPTYNODES and LEMPTYNODES of the apexes. It then pushes the element requested to be inserted down the banyan heap using operation *insert-adjust*. The operation *insert-adjust* pushes down the element (along the paths from the root of the partition to the bases) until it finds its correct position.

Upon receiving *INSERT*(p,(item,priority)) by processor p_1, it executes the following codes. The letter p refers to the address of the leftmost apex and (item, priority) is the pair requested to be inserted.

```
found ← false
While (not found ) do
 if DATA(p) ≠ null then
  if priority > PRIORITY(p) then
   begin
    if LEMPTYNODES(p)≠ 0 or
       REMPTYNODES(p)≠ 0 then
     begin
      if LEMPTYNODES(p) >
        REMPTYNODES(p) then
        begin
          p' ← RCHILD(p);
          REMPTYNODES(p) ←
          REMPTYNODES(p) −1
        end
      else
        begin
          p' ← LCHILD(p);
          LEMPTYNODES(p) ←
            LEMPTYNODES(p) −1
        end
      send(P2 , 'insert-adjust(p', DATA(p))');
      DATA(p) ← item;
      PRIORITY(p)← priority;
      found ← true
     end
    else
      p ← NEXT(p)
   end
  else
   begin ( priority < PRIORITY(p) )
    if LEMPTYNODES(p) ≠ 0 or
      REMPTYNODES(p)≠ 0 then
      begin
        if LEMPTYNODES(p) >
           REMPTYNODES(p) then
        begin
          p' ← LCHILD(p);
          LEMPTYNODES(p) ←
            LEMPTYNODES(p) −1
        end
        else
        begin
          p' ← RCHILD(p);
          REMPTYNODES(p) ←
            REMPTYNODES(p) −1
        end
        send(p2, 'insert-adjust(p', (item,priority))');
        found ← true
      end
    else
      p ← NEXT(p)
   else
    begin
      DATA(p) ← item;
      PRIORITY(p) ← priority
    end;
```

Processor P_i, $(2 \leq i \leq L)$, upon receiving *insert-adjust*(p,(item,priority)) executes the following codes.

```
if DATA(p) ≠ null then
  if priority > PRIORITY(p) then
    begin
      if LEMPTYNODES(p) ≠ 0 or
        REMPTYNODES(p) ≠ 0 then
        begin
          if LEMPTYNODES(p) >
            REMPTYNODES(p) then
            begin
              p' ← LCHILD(p) ;
              LEMPTYNODES(p)←
                LEMPTYNODES(p) −1
            end
          else
            begin
              p' ← RCHILD(p);
              REMPTYNODES(p) ←
                LEMPTYNODES(p) −1
            end
        end
        send(p_{i+1},'insert-adjust(p'
          ,(DATA(p),PRIORITY(p)))');
        DATA(p) ← item;
        PRIORITY(p) ← priority
    end
  else
    begin
      if LEMPTYNODES(p) ≠ 0 or
        REMPTYNODES(p) ≠ 0 then
        begin
          if LEMPTYNODES(p) >
            REMPTYNODES(p) then
            begin
              p' ← RCHILD(p);
              REMPTYNODES(p) ←
                REMPTYNODES(p) −1
            end
          else
            begin
              p'← LCHILD(p);
              LEMPTYNODES(p) ←
                LEMPTYNODES(p) −1
            end
        send(p_{i+1}, 'insert-adjust(p'
          ,(item,priority))');
        end
      else
        begin
          DATA(p) ← item;
          PRIORITY(p) ← priority
        end
  else
    begin
      DATA(p) ← item;
      PRIORITY(p) ← priority
    end;
```

XMAX operation first locates the apex which contains the element with the highest priority, reports that element to the outside world, and then fills up that apex with the element in one of its children. *xmax-adjust* is responsible for restructuring the banyan as it moves down the heap. When *XMAX*(p) is received by processor p_1, it executes the following codes,where p is the address of the leftmost apex. This address is known to the outside world (front end computer).

```
p' ← p
while NEXT(p) ≠ nil and
  DATA(NEXT(p)) ≠ null do
  begin
    if PRIORITY(p) > PRIORITY(NEXT(p)) then
      p' ← p
    p ← NEXT(p)
  end
  send('outside world', DATA(p'));
  receive (p_2, ((PRIORITY(RCHILD(p'))
      ,DATA(RCHILD(p'))),
      (PRIORITY(LCHILD(p'))
      ,DATA(LCHILD(p')))) )
  if DATA(RCHILD(p')) ≠ null or
    DATA(LCHILD(p'))≠ null then
    if DATA(RCHILD(p')) >
      DATA(LCHILD(p')) then
      begin
        DATA(p') ← DATA(RCHILD(p'));
        PRIORITY(p') ←
          PRIORITY(RCHILD(p'));
        REMPTYNODES(p') ←
          REMPTYNODES(p') + 1;
        send(p_2, 'xmax-adjust(RCHILD(p')')
      end
    else
      begin
        DATA(p') ← DATA(LCHILD(p'));
        PRIORITY(p') ←
          PRIORITY(LCHILD(p'));
        LEMPTYNODES(p') ←
          LEMPTYNODES(p') + 1;
        send (p_2, 'xmax-adjust(LCHILD(p')')
      end
  else
    begin
      DATA(p') ← null;
      PRIORITY(p') ← −1
    end
```

Processor p_i, $(2 \leq i \leq L)$, upon receiving *xmax-adjust*(p)) executes the following codes.

```
receive(p_{i+1}, ((PRIORITY(RCHILD(p))
      ,DATA(RCHILD(p))),
      (PRIORITY(LCHILD(p))
      ,DATA(LCHILD(p)))) );
if DATA(RCHILD(p)) ≠ null or
      DATA(LCHILD(p)) ≠ null then
  if DATA(RCHILD(p)) > DATA(LCHILD(p)) then
    begin
      DATA(p) ← DATA(RCHILD(p));
      PRIORITY(p) ← PRIORITY(RCHILD(p));
      REMPTYNODES(p) ←
          REMPTYNODES(p) + 1;
```

```
      send(p_{i+1}, 'xmax-adjust(RCHILD(p))')
    end
  else
    begin
      DATA(p) ← DATA(LCHILD(p));
      PRIORITY(p) ← PRIORITY(LCHILD(p));
      LEMPTYNODES(p) ← PRIORITY(p) + 1;
      send(p_{i+1}, 'xmax-adjust(LCHILD(p))')
    end
else
  begin
    DATA(p) ← null;
    PRIORITY(p) ← −1
  end
```

Remark 2 The elements stored in the apex nodes are not ranked in any particular order. This speeds up the insertion process, but will lead to O(M) time for deletion. It is possible to insert the elements in such a way that the element with the highest priority is always available at the leftmost apex, in this case, locating the correct apex to initiate the insertion takes O(M) time. This method seems to be more efficient because a portion of the time spent to find the correct position can be overlapped with the time spent to locate an apex with zero REMPTYNODES or zero LEMPTYNODES. In the algorithms presented above we have used the first approach. The latter approach will be reported in another paper.

From the properties of *SW*-banyan graphs and the above algorithms we can state the following results. For proofs of the lemmas refer to [29].

Lemma 1 *a) The insert-adjust operation never encounters a node which is non-null and has zero LEMPTYNODES and zero REMPTYNODES. b) The insert-adjust operation always finds a null node to insert its element.*

Remark 3 Deletion of an element from partition i may cause one of the elements in other partitions whose nodes are reachable from apex i to become null. This happens if a delete operation causes the *xmax-adjust*, on its way down the heap, to move up the content of one of the leaf nodes of partition i to fill up its parent which has been emptied by *xmax-adjust* operation at the previous step. The emptiness of this node now will be reflected in the REMPTYNODES or LEMPTYNODES of apex i. This node is now reachable by partition from apex i and will be filled by an insertion initiated at apex i. The maximum number of nodes that may become reachable by partition from apex i as a result of a deletion is equal to $(L+1)-D$ where D is the depth of partition i.

Lemma 2 *Zero REMPTYNODES and zero REMPTYNODES for an apex does not imply that all the nodes in the corresponding partition are non- null.*

Lemma 3 *Apex i, $(1 \leq i \leq N)$, always contains the element which has the highest priority among the elements stored in the nodes of partition i.*

Lemma 4 *The element with the highest priority is always reported by operation* XMAX.

Definition 10 *A partition induced by LEMPTYNODES and REMPTYNODES fields of apex i is the set of all nodes which are reachable by partition from apex i.*

5 Retrieval at Percentile Levels

One of the advantages of banyan heap machine over other machines is in its ability to retrieve elements at different percentile levels. In this section we derive formulas for the percentile level of the element reported by operation *XMAX* for different cases.

Definition 11 *An element removed from a banyan heap is at percentile c if at least c percent of the elements stored in the heap have priority less than or equal to the priority of the deleted element.*

We define $REMPTYNODES_i$ and $LEMPTYNODES_i$ to denote respectively the value of REMPTYNODES field and LEMPTYNODES field of apex i. The proof of the following 4 lemmas are immediate from the definitions of REMPTYNODES and LEMPTYNODES.

Lemma 5 *The total number of null nodes which are reachable by partition from apex i is $REMPTYNODES_i + LEMPTYNODES_i$.*

Lemma 6 *If an MxM partitioned rectangular SW-banyan banyan heap is full up to apex d then*

$$\sum_{j=1}^{d}(REMPTYNODES_j + LEMPTYNODES_j) = 0.$$

Lemma 7 *In an MxM rectangular SW-banyan, the total number of null nodes reachable by partition from apexes 1 through d, written NULLNODES(M,d), is given by:*

$$NULLNODES(M,d) =$$

$$\sum_{i=1}^{d}(REMPTYNODES_i + LEMPTYNODES_i) + K.$$

where K is the number of null apexes i, $(i \leq d)$.

Lemma 8 *The total number of non-null nodes in a MxM partitioned rectangular SW-banyan, written NONNULLNODES(M,M), is $M(\log M + 1) - NULLNODES(M,M)$.*

Lemma 9 *The total number of partitions of depth k in a full partitioned banyan heap up to apex d, written NP_k, is given by:*

$$NP_k = \lfloor \frac{d - \sum_{j=1}^{k-1} NP_j}{2} \rfloor$$

where $NP_1 = \lfloor \frac{d}{2} \rfloor$.

Lemma 10 *The total number of non-null nodes in a full MxM partitioned rectangular SW-banyan up to apex d, written size(M,d), is given by:*

$$size(M,d) = \sum_{k=0}^{M \log M} 2^k + \sum_{j=1}^{d} NP_j 2^j$$

Lemma 11 *If an MxM rectangular SW-banyan partitioned heap is full up to apex d then the element stored at apex 1 is at percentile level*

$$\frac{(2M-1) * 100}{size(M,d)}.$$

Lemma 12 *In an MxM partitioned rectangular SW-banyan heap which is full up to apex d, if operation XMAX investigates i,(i<d), non-null apexes then the percentile of the reported element is*

$$\frac{size(M,i) * 100}{size(M,d)}.$$

Lemma 13 *If operation XMAX examines apexes 1 through d in an MxM rectangular banyan heap then the percentile of the reported element is smaller than or equal to*

$$\frac{size(M,d) * 100}{(size(M,M) - NULLNODES(M,M))}.$$

Remark 4 A partition banyan heap can be converted into a banyan heap by an operation called *adjust*. $M \log M - 2$ *adjust* operations are broadcast by *XMAX* operation when it inserts an element into an empty partition. These *adjust* operations cause some the elements in the nodes of those partitions which are reachable from apex i to move up and fill up the nodes of partition i. As a result, all the nodes whose contents (empty or non-empty) have been moved by *adjust* operation become reachable by partition from apex i. It should be noted that some of the *adjust* operation initiated at processor p_1 by *XMAX* operation may not have any effect on the structure of the heap. The advantage of banyan heap over partitioned banyan heap is that it allows a more uniform distribution of data elements among the partition in the heap and leads to a more uniform increase in the percentile level of the reported element as the number of examined apexes is increased. Algorithms for *XMAX*, *xmax-adjust* and *insert-adjust* are the same for banyan heap. The operation *INSERT* and the new operation *adjust* are described in details in [29].

6 Conclusion

A novel design based on MxM *SW*-banyan heap data structure, called banyan heap machine, for performing a group of priority queue operations on a set of elements is presented. This machine is a linear array of $\log M + 1$ processors (one processor for each level of banyan) which receives a stream of priority queue operations and executes them in a piplined manner. Unlike some of the existing designs, processors in banyan heap machine do not have geometrically varying size memory. This results in a completely homogeneous system. The response time for *XMAX*, and pipeline period for both the *XMAX* and *INSERT* operations is $O(1)$, independent of the length of the array of processors. However, it takes $O(M)$ time for each of the *XMAX* or *INSERT* operation to be executed. With banyan heap machine, it is possible to retrieve elements at different percentile levels.

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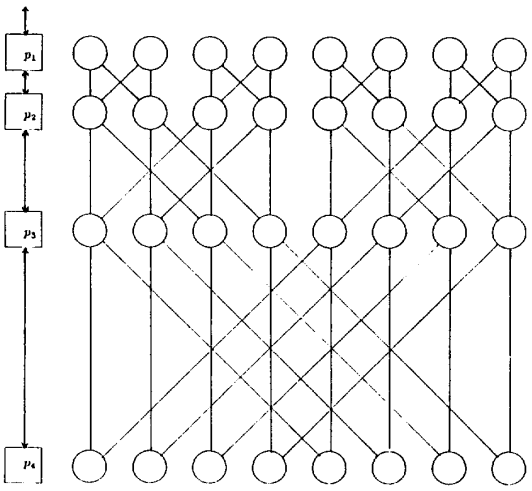

Banyan Heap Machine

Figure 1

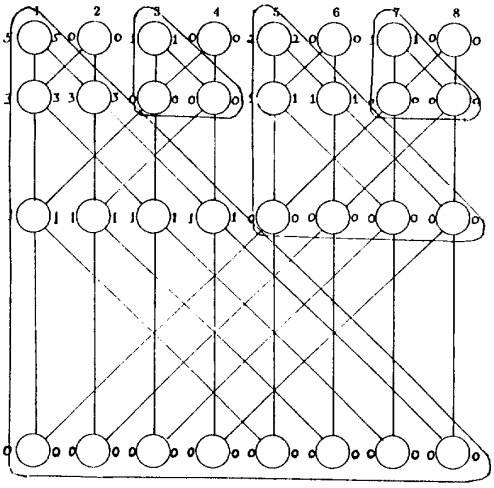

Figure 2