Coordinating a three level supply chain with flexible return policies

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Abstract

This paper studies the coordination issue of a three level supply chain selling short life cycle products in a single period model. The manufacturer first negotiates the trade contract with the retailer, then with the supplier. We construct the so-called flexible return policy by setting the rules of pricing while postponing the determination of the final contract prices. With return policies between each pair of adjacent firms, unsold products or used modules dissembled from the unsold products are returned level by level from the retailer to the upstream firms and each firm shares in the loss due to the overstock. We show that the three level supply chain can be fully coordinated with appropriate contracts and the total profit of the channel can be allocated with any specified ratios among the firms.

Keywords: Supply chain management; Return policy; Coordination

1. Introduction

Supply chain is a network of suppliers, manufacturers, distributors, retailers and customers. When a supply chain is decentralized controlled, two well-known reasons will lead to inefficient performance of the system. The first reason is information asymmetry. A vivid instance is the famous bullwhip effect. “Distorted information from one end of a supply chain to the other can lead to tremendous inefficiencies” [1]. The second reason is double marginalization, occurs whether or not information asymmetry exists and “whenever the supply chain’s profits are divided among two or more firms” [2], which apparently originated in the economics literature [3].

Contracts are effective instruments to conquer information asymmetry and double marginalization. During the last two decades, a growing interest has been devoted to the supply chain coordination issues. A large body of literature has explored how to coordinate supply chains with all kinds of popular contracts, such as return policies [4,5], quantity discounts [6,7], quantity flexibility contracts [8,9], backup agreements [10], revenue sharing contracts [11,12], sales rebate contracts [13], options [14], and so on. We refer the reader to [15,16] for more detailed surveys of contracts.

Most of the existing literature studied channel coordination in two level supply chain environment. For example, Pasternack [5] claimed that an appropriate return policy can fully coordinate a single-supplier single-retailer supply chain, which was then extended by Mantrala and Raman [17] to the situation where the retailer has several stores. Ding and Chen [18] considered the case where an assembler who also acts as a retailer faces two complementary suppliers. The return policies between each supplier and the assembler react
on each other and happen to fully coordinate the whole system in equilibrium.

However, in reality, supply chains often contain three or even more levels of firms. Coordination should not be limited within only two levels. Firms (e.g., HP, Lenovo, Nokia) procure modules from their suppliers and sells various fashion electronic products through their retailers. To encourage the retailers to order more products to improve the service level for their customers, one of the feasible solutions for the manufacturer is to offer return policies to his retailers. On the other hand, the suppliers might also want to offer return policies to the manufacturer for the same reason. A natural idea is that, if we can make these contracts work cooperatively, we may develop the competitive potentials of the supply chains as much as possible. This paper extends previous works by adding a third tier to the supply chain and explores if incorporating return policies from both end of the supply chain (i.e., supplier–manufacturer and manufacturer–retailer) can fully coordinate three level supply chains.

In two level supply chain coordination models such as Pasternak [5], the cost structure of the contract-offering firm (e.g., a supplier) is assumed to be known before signing the contract. Hence, the best contract that coordinating the channel can be determined at once. While in a three level demand-driven supply chain consisting of a supplier, a manufacturer and a retailer, the trade contract between the manufacturer and the retailer may be negotiated first as assumed in our model, and under some circumstances, the manufacturer may not be able to precisely anticipate the contract prices between the supplier and himself which is assumed to negotiate later. Therefore, the cost structure of the manufacturer may have not been confirmed when he first negotiates the contract with the retailer. The manufacturer and the retailer have to make their decisions based on estimation. As a result, channel coordination can only be achieved by little chance.

This paper studies how to fully coordinate a three level supply chain with return policies. We construct the so-called flexible return policy by setting the rules of pricing while postponing the determination of the final contract prices. With such flexible contract between the manufacturer and the retailer and a return policy between the supplier and the manufacturer, unsold products or used modules disassembled from the unsold products are returned level by level from the retailer to the upstream firms and each firm shares in the loss due to the overstock. We show that the three level supply chain can be fully coordinated by appropriate contract prices and the total profit of the channel can be allocated with any specified ratios among the firms.

Some papers have made similar research attempts. Lee [19] investigated return policy in a distribution channel consisting of a supplier, a retailer and a discount sales outlet (DSO) in which the residual products after the selling season are moved to DSO for a permanent clearance sale. The DSO is a complementary sales agent to the retailer and the channel structure considered is distinct from ours.

Munson and Rosenblatt [20] considered a three level supply chain and explored the benefits of using quantity discounts on both end of the supply chain to decrease costs from the view of the manufacturer. Our model differs from theirs in several aspects besides the different types of contracts involved. We consider a single period model based on the classic newsvendor model, in which the channel sells short life cycle products with stochastic demand at a fixed price, and the coordination is achieved by stimulating the retailer to order more. While Munson and Rosenblatt [20] studied a multiple period inventory model based on the classic EOQ model, in which the channel sells commodities with deterministic demand, and their coordination is achieved by saving setup cost and holding cost.

The rest of this paper is organized as follows. Section 2 describes the basic settings of the model; Section 3 analyzes two benchmark scenarios, i.e., scenario C in which the whole channel is centralized controlled and scenario D in which the firms are decentralized controlled with price-only contracts; Section 4 studies how to coordinate the channel by return policies. Again, we consider two scenarios, i.e., scenario P in which the supplier offers price-only contract to the manufacturer while the latter offers return policy to the retailer, and scenario B in which both the supplier and the manufacturer offer return policies to their corresponding downstream firms. Finally Section 5 concludes the paper.

2. Basic settings

Consider a three level supply chain consisting of a supplier, a manufacturer and a retailer (or a manufacturer, a distributor and a retailer, etc.). All three firms are assumed to be risk neutral and pursue expected profit maximization. See Fig. 1 for the diagram of the considered three level supply chain. The retailer sells short life cycle products, such as personal computers, consumer electronics or fashion items, with highly uncertain demand. The products are sold only in one period. As the lead times of such goods are much longer than their
selling seasons, therefore, the firms have no chance to place a second order.

The sequence of events is as follows:

1. The retailer forecasts the demand, negotiates his trade contract with the manufacturer and places his order;
2. The manufacturer negotiates another trade contract with the supplier and passes the order to the latter;
3. The supplier procures raw materials, produces basic modules and forward them to the manufacturer;
4. The manufacturer embeds some key components into the basic modules (or any other kind of value-added process) and delivers the final products to the retailer. For simplicity, assume that one final product contains one basic module and one key component;
5. When the selling season arrives, the retailer sells the products at a fixed price in the market. Any unmet demand incurs goodwill cost to the retailer;
6. After the selling season, the retailer returns the residual products to the manufacturer if the latter offers a return policy;
7. If the supplier offers a return policy, the manufacturer disassembles the residual products into used modules and used components, and returns the used modules to the supplier;
8. All the residual products, used modules and used components, regardless which firm holds them, are salvaged.

Only two kinds of trade contracts are considered in this paper, i.e., the price-only contract, with which the firms trade at one transfer price for all units without proviso, and the return policy, with which the firms trade at one transfer price for all units while trade reversely with partial refund for unlimited residual units.

The following are exogenous variables used in the model:

- \( c \) supplier’s procurement cost of raw material for unit product
- \( c^S \) supplier’s processing cost of unit basic module
- \( c^M \) manufacturer’s value-added cost of unit product
- \( c^R \) retailer’s treating cost of unit product
- \( p \) retailer’s fixed sales price of unit final product
- \( g \) retailer’s goodwill cost for unit unmet demand
- \( s^S \) salvage value of unit used module
- \( s^M \) salvage value of unit used component
- \( s \) salvage value of unit residual product and \( s = s^S + s^M \)
- \( D \) positive stochastic customer demand
- \( \mu \) mean of the customer demand \( D \)
- \( \sigma \) standard deviation of the customer demand \( D \)
- \( F \) differentiable cumulated distribution function of the customer demand \( D \)
- \( f \) probability distribution function of the customer demand \( D \).

And the following are price-decision variables:

- \( w^M \) wholesale price of unit product offered by the manufacturer
- \( b^M \) return price of unit residual product offered by the manufacturer
- \( w^S \) transfer price of unit basic module offered by the supplier
- \( b^S \) return price of unit used module offered by the supplier.

More notation will be defined later when needed. In order to avoid trivial cases, we assume

\[
\begin{align*}
    s^S &< c + c^S < w^S \\
    s^S &\leq b^S < w^S \\
    s^M &< c^M < w^M - w^S \\
    s^M + b^S &\leq b^M < w^M < p - c^R.
\end{align*}
\]

These inequalities ensure that each firm makes positive profit and the chain will not produce infinite products.

3. Benchmarks

3.1. The centralized supply chain

There is no doubt that the supply chain performs best when all three firms are centralized controlled (scenario
C. It is a classic newsvendor problem. When the channel decides to order $Q_C$ units of products for sale, the total profit of the channel, $\Pi_T^C$ (T stands for the total supply chain and C for scenario C, similar usage hereafter), is

$$\Pi_T^C(Q_C) = p \cdot (Q_C \land D) + s \cdot (Q_C - D)^+ - g \cdot (D - Q_C)^+ - (c + c^S + c^M + c^R) \cdot Q_C$$

in which $(x \land y) = \min(x, y)$, and $x^+ = \max(x, 0)$.

We can obtain the unique optimal order quantity of the channel by using the first and second order conditions,

$$Q_C^* = F^{-1}\left(\frac{p + g - c - c^S - c^M - c^R}{p + g - s}\right),$$

where $F^{-1}(y) = \inf\{x : F(x) = y\}$. Define

$$K = \frac{p + g - c - c^S - c^M - c^R}{p + g - s}$$

as the ideal critical ratio of the system, then $Q_C^* = F^{-1}(K)$.

The profit of the whole channel in our model only depends on the order quantity. Thus, if and only if ordering $Q_C^*$, the total expected profit of the supply channel is maximized at

$$E\Pi_T^C(Q_C^*) = (p + g - s) \cdot \int_0^{Q_C^*} x \, dF - g \cdot \mu.$$

3.2. The decentralized system with price-only contracts

Consider scenario D in which the manufacturer and the retailer decide to implement a price-only contract. The supplier and the manufacturer then have no choice but to implement price-only contract too as the manufacturer would have nothing to return to the supplier.

Assume that the contracts prices here are negotiated based on the firms’ bargaining powers, which are set at $w_D^M$ and $w_D^S$ finally. Certainly, the prices should satisfy (1). When the retailer decides to order $Q_D$ units of products for sale, his profit is

$$\Pi_D^R(Q_D) = p \cdot (Q_D \land D) + s \cdot (Q_D - D)^+ - g \cdot (D - Q_D)^+ - (c^R + w_D^M) \cdot Q_D.$$ 

Similar to (2), the retailer will order as many as

$$Q_D^* = F^{-1}\left(\frac{p + g - c^R - w_D^M}{p + g - s}\right),$$

which is also the final order quantity of the channel as the manufacturer has no willing to order more. Easy to verify that $Q_D^* < Q_C^*$, and $E\Pi_D^R(Q_D^*) < E\Pi_T^C(Q_C^*)$ where $E\Pi_T^C(Q_C^*)$ denotes the total channel profit in scenario D. System inefficiency exhibits as long as the supplier and/or the manufacturer price above their costs, which is the only way for them to make positive profits. The upstream firms have to provide incentive mechanism such as return policy to encourage the retailer to order more to improve the performance of the firms and of the channel.

4. Coordinating with flexible return policies

4.1. The difficulty of full coordination

Now we turn to the case in which the manufacturer offers return policy to the retailer. Obviously, the best contract between the manufacturer and the retailer depends on the cost structure of the manufacturer, and therefore depends on the contract between the supplier and the manufacturer. Assume that the supplier and the manufacturer implement a trade contract with prices $w^S$ and $b^S$ ($b^S = s^S$ for price-only contract and $b^S > s^S$ for return policy), then the best decisions for the manufacturer and the retailer are to select $w^M$ and $b^M$ to fully coordinate themselves and allocate their total profit properly.

When the retailer decides to order $Q$ units of products, his profit is

$$\Pi_D^R(Q) = p \cdot (Q \land D) + b^M \cdot (Q - D)^+ - g \cdot (D - Q)^+ - (c^R + w^M) \cdot Q.$$ 

So, the best order quantity for the retailer is

$$Q_R = F^{-1}\left(\frac{p + g - c^R - w^M}{p + g - b^M}\right).$$ (4)

While the total profits of the manufacturer and the retailer are

$$\Pi_D^M(Q) = p \cdot (Q \land D) + (b^S + s^M) \cdot (Q - D)^+ - g \cdot (D - Q)^+ - (c^M + c^R + w^S) \cdot Q.$$ 

Hence, the optimal order quantity for the total profits of these two firms are

$$Q_M^R = F^{-1}\left(\frac{p + g - c^R - w^M}{p + g - b^S}\right).$$

In order to achieve full coordination of the manufacturer and the retailer, they need to set the contract prices to satisfy $Q_R = Q_M^R$, or equivalently,

$$\frac{p + g - c^R - w^M}{p + g - b^M} = \frac{p + g - c^R - w^S}{p + g - s^M - b^S}. $$ (5)
There are infinite pairs of $w^M$ and $b^M$ satisfy (5), and each pair divides the total profits of the two firms differently. One more profit-sharing term is required to determine the final contract prices and such term should ensure that each firm gets more when their total profits increase. Again, we assume that the determination of the final contract prices are based on negotiation.

In order to fully coordinate of the whole channel, we require $Q^R = Q^C_e$. The goal is achieved if and only if the contract prices between the supplier and the manufacturer satisfy $Q^{MR} = Q^e_C$, i.e.,

$$\frac{p + g - c^M - c^R - w^S}{p + g - s^M - b^S} = \frac{p + g - c - c^S - c^M - c^R}{p + g - s} = K. \quad (6)$$

As we state in the event sequence, the trade contract between the manufacturer and the retailer are negotiated first. They face two problems. The first problem is that they can not precisely anticipate the negotiated price-only contract to the manufacturer, the flexible return policy between them. In other words, channel coordination can only be achieved by little chance.

4.2. The flexible return policy

We construct the so-called flexible return policy by just setting the rules of pricing while postponing the determination of the final contract prices. Assume that the manufacturer and the retailer decide to implement a flexible return policy, all they need to do is to declare that $w^M$ and $b^M$ will be determined based on (5) along with a certain profit-sharing term, and the order quantity is determined according to (4). Such pricing rules can ensure that the manufacturer and the retailer are always fully coordinated no matter what contract prices the supplier and the manufacturer agree on later. The final contract prices between the manufacturer and the retailer and the order quantity of the retailer are not settled until the contract prices between the supplier and the manufacturer has been settled.

The supplier and the manufacturer then have to negotiate their trade contract. If they adopt a price-only contract with transfer price $w^S_p$ (scenario P), only the manufacturer and the retailer are coordinated, and the final order quantity is

$$Q^*_p = F^{-1}\left(\frac{p + g - c^M - c^R - w^S_p}{p + g - s}\right). \quad (7)$$

Easy to verify that $Q^*_p < Q^*_C$ and $\Pi^{T}_P(Q^*_p) < \Pi^{T}_C(Q^*_C)$ where $\Pi^{T}_P$ denotes the total channel profit in scenario P. If $w^S_D \leq w^S_D$, we also have $Q^*_p < Q^*_p$ and $\Pi^{T}_D(Q^*_D) < \Pi^{T}_D(Q^*_D)$. When the supplier offers price-only contract to the manufacturer, the flexible return policy between the manufacturer and the retailer can only partially mitigate the double marginalization, and full channel coordination can never be achieved.

Hereafter we concentrate on the case in which the supplier also offers return policy to the manufacturer (scenario B). Denote $Q^*_B$ as the optimal order quantity of the retailer in Scenario B.

The determination of the contract prices between the supplier and the manufacturer can be thought of following two steps. First, the two firms decide the ratio in the right hand side of (5) and in fact decide the order quantity and the total profit of the channel. Then they modulate the contract prices based on the decided ratio in the first step to properly divide the total channel profit into two parts, one for the supplier and the other for the manufacturer and the retailer. Certainly, such allocation should ensure that when the channel profit increases, the supplier gets more, the manufacturer and the retailer totally get more, and hence both the manufacturer and retailer get more according to the profit-sharing term in the flexible return policy between them. In other words, both the profit of the supplier and the profit of the manufacturer are increasing functions of the total channel profit. When the supplier and the manufacturer contrive to satisfy $Q^{MR} = Q^*_C$, the total profit of the channel is maximized, and so do their own profits. The optimal order quantity is $Q^*_B = Q^*_C$ and the channel is fully coordinated.

As a rule, a flexible return policy generally consists of two terms to set the contracts prices. A critical ratio term to determine the total profits of related firms, and a profit-sharing term to allocate the profit properly among the firms.

4.3. The optimal prices of full coordination

The whole channel is fully coordinated as long as (5) and (6) are satisfied. Define the equivalent effective
sales quantity of the coordinated system as

$$\lambda = \frac{\int_0^{Q_B} x \, dF}{F(Q_B^*)},$$  \hfill (8)

then

$$\lambda = \frac{F(Q_B^*) \cdot Q_B^* - \int_0^{Q_B^*} F(x) \, dx}{F(Q_B^*)} < Q_B^*.$$

Since

$$\frac{\partial \lambda}{\partial Q_B^*} = \frac{f(Q_B^*) \cdot (Q_B^* - \lambda)}{F(Q_B^*)} > 0,$$

we have

$$\lambda = \lambda(Q_B^*) < \lambda(\infty) = \mu.$$  \hfill (9)

According to (5), (6) and (8), and note that $F(Q_B^*) = F(Q_C^*) = K$, we can derive that

$$E \left[ (Q_B^* \wedge D) + (Q_B^* - D)^+ \cdot \left(1 - \frac{1}{K}\right) \right]$$

$$= \int_0^{Q_B^*} x \, dF + \int_{Q_B^*}^{\infty} Q_B^* \, dF$$

$$+ \left(1 - \frac{1}{K}\right) \cdot \int_0^{Q_B^*} (Q_B^* - x) \, dF$$

$$= \frac{1}{K} \cdot \int_0^{Q_B^*} x \, dF = \frac{\int_0^{Q_B^*} x \, dF}{F(Q_B^*)} = \lambda,$$  \hfill (10)

which indicates that $\lambda$ combines the weighted returned quantity $(Q_B^* - D)^+$ with the sales quantity $(Q_B^* \wedge D)$. That’s why we call $\lambda$ the equivalent effective sales quantity. As $(Q_B^* \wedge D) + (Q_B^* - D)^+ = Q_B^*$, (10) can also be written as

$$E \left[ Q_B^* - \frac{1}{K} \cdot (Q_B^* - D)^+ \right] = \lambda.$$  \hfill (11)

Define the equivalent effective sales price of the coordinated system as

$$\rho = \frac{p \cdot \lambda - g \cdot (\mu - \lambda)}{\lambda},$$  \hfill (12)

in which $p \cdot \lambda$ is the equivalent sales revenue, $g \cdot (\mu - \lambda)$ is the goodwill cost as $(\mu - \lambda)$ is the equivalent unmet demand. So $\rho$ incorporates the goodwill cost into the sales price.

Denote

$$\delta^S = w^S - c - c^S$$
$$\delta^M = w^M - w^S - c^M$$
$$\delta^R = \rho - c^R - w^M$$
$$\delta^T = \delta^S + \delta^M + \delta^R$$  \hfill (13)

as the equivalent marginal sales revenues of the three firms and of the channel. Then, based on (10)–(13), the expected profit of the three firms are

$$E \Pi^R_B(Q_B^*) = E \left[ p \cdot (Q_B^* \wedge D) + b^M \cdot (Q_B^* - D)^+ - g \cdot (D - Q_B^*)^+ - (c^R + w^M) \cdot Q_B^* \right]$$

$$= E \left[ (p + g - c^R - w^M) \cdot (Q_B^* \wedge D) + (b^M - c^R - w^M) \cdot (Q_B^* - D)^+ - g \cdot D \right]$$

$$= \left( p + g - c^R - w^M \right) \cdot E \left[ (Q_B^* \wedge D) + (1 - \frac{1}{K}) \cdot (Q_B^* - D)^+ \right] - E[g \cdot D]$$

$$= \left( p + g - c^R - w^M \right) \cdot \lambda - g \cdot \mu = \delta^R \cdot \lambda.$$  \hfill (14)

$$E \Pi^M_B(Q_B^*) = E \left[ (w^M - w^S - c^M) \cdot Q_B^* - (b^M - s^M - b^S) \cdot (Q_B^* - D)^+ \right]$$

$$= (w^M - w^S - c^M)$$

$$\cdot E \left[ Q_B^* - \frac{1}{K} \cdot (Q_B^* - D)^+ \right]$$

$$= (w^M - w^S - c^M) \cdot \lambda = \delta^M \cdot \lambda.$$  \hfill (15)

$$E \Pi^C_B(Q_B^*) = E \left[ (w^S - c - c^S) \cdot Q_B^* - (b^S - s^S) \cdot (Q_B^* - D)^+ \right]$$

$$= (w^S - c - c^S)$$

$$\cdot E \left[ Q_B^* - \frac{1}{K} \cdot (Q_B^* - D)^+ \right]$$

$$= (w^S - c - c^S) \cdot \lambda = \delta^S \cdot \lambda.$$  \hfill (16)

That is to say, when (5) and (6) are both satisfied, the fully coordinated supply chain can be regarded as if the channel sales exactly $\lambda$ units of products at price $\rho$. The original supply chain is transformed to a brand-new equivalent system in which there is no need to consider residual products and goodwill cost anymore.

To obtain the optimal contracts prices, we have to detail the profit-sharing terms in the two contracts.
Assume that the manufacturer and the retailer adopt a simple rule that the two firms allocate the profit according to some negotiated ratios,

\[ EII^M : EII^R = \xi^M : \xi^R, \quad \xi^M, \xi^R > 0 \quad (17) \]

and so do the supplier and the manufacturer,

\[ EII^S : (EII^M + EII^R) = \xi^S : (\xi^M + \xi^R), \quad \xi^S > 0, \quad (18) \]

where the factors \( \xi^S, \xi^M \) and \( \xi^R \) are normalized to satisfy \( \xi^S + \xi^M + \xi^R = 1 \). Based on (14)–(16),

\[ EII^S(Q_B^*) : EII^M(Q_B^*) : EII^R(Q_B^*) = \delta^S : \delta^M : \delta^R. \quad (19) \]

From (17) to (19), we have

\[ \frac{\delta^S}{\xi^S} = \frac{\delta^M}{\xi^M} = \frac{\delta^R}{\xi^R} = \frac{\delta^S + \delta^M + \delta^R}{\xi^S + \xi^M + \xi^R} = \frac{\delta^T}{1}. \quad (20) \]

Solve (5), (6) and (20) together, we get the optimal contract prices

\[
\begin{align*}
(w^S)^* &= c + c^S + \xi^S \cdot \delta^T \\
(b^S)^* &= s^S + \frac{\xi^S}{K} \cdot \delta^T \\
(w^M)^* &= c + c^S + c^M + (\xi^S + \xi^M) \cdot \delta^T \\
(b^M)^* &= s + \frac{(\xi^S + \xi^M)}{K} \cdot \delta^T.
\end{align*}
\]

The optimal transfer prices are more intelligible in the transformed equivalent system. For example, the marginal cost of the supplier, \( c + c^S \), plus his allocated equivalent marginal sales revenue, \( \xi^S \cdot \delta^T \), gives his optimal transfer price.

The above analysis shows that the total profit of the channel can be shared with any specified ratios among the firms. The profit allocated ratios can be thought of the firm’s bargaining powers. A simple, effective but not exclusive way to set the profit allocated ratios is to let them equal to the profit ratios between the corresponding firms before introducing the contracts, which can certainly ensure that each firm earns more as the contracts increase the total profit of the channel.

**Proposition.** With appropriate flexible return policy between the manufacturer and the retailer, and a return policy between the supplier and the manufacturer, the whole channel can be fully coordinated and the total profit of the channel can be shared with any specified ratios among the firms.

![Fig. 2. Marginal effect of the return policies.](https://example.com/fig2.png)

### 4.4 Effect of the flexible return policy

The expected channel profit in scenario D is

\[
EII_D^T(Q_D^*) = E \left[ p \cdot (Q_D^* \land D) + s \cdot (Q_D^* - D)^+ \right. \\
- g \cdot (D - Q_D^*)^+ \\
- \left. \left( c + c^S + c^M + c^R \right) \cdot Q_D^* \right]
\]

\[
= E \left[ \left( p + g - c - c^S - c^M - c^R \right) \cdot Q_D^* \\
- (p + g - s) \cdot (Q_D^* - D)^+ - g \cdot D \right]
\]

\[
= (p + g - s) \cdot \left[ F(Q_B^*) \cdot Q_B^* \\
- \int_0^{Q_B^*} F(x) \, dx \right] - g \cdot \mu
\]

\[
= (p + g - s) \cdot \text{AREA}_A - g \cdot \mu,
\]

where

\[
E[(Q_D^* - D)^+] = \int_0^{Q_D^*} (Q_D^* - x) \, dF
\]

\[
= (Q_B^* - x) \cdot F(x)|_0^{Q_B^*} \\
+ \int_0^{Q_B^*} F(x) \, dx
\]

\[
= \int_0^{Q_B^*} F(x) \, dx
\]

and \( \text{AREA}_A \) is the area of region A in Fig. 2.

When the manufacturer and the retailer implement a flexible return policy while the supplier offers price-only contract to the manufacturer as considered in
The expected channel profit becomes

\[ EII_b^2(Q^*_b) = E \left[ p \cdot (Q^*_b \land D) + s \cdot (Q^*_b - D) \right] + g \cdot (D - Q^*_b)^+ - \left( c + c^S + c^M + c^R \right) \cdot Q^*_b \]

\[ = (p + g - s) \cdot \left[ F(Q^*_b) \cdot Q^*_b - \int_0^{Q^*_b} F(x) \, dx \right] - g \cdot \mu \]

\[ = (p + g - s) \cdot \text{AREA}_{ABC} - g \cdot \mu, \]

where AREA_{ABC} is the total area of regions A, B and C in Fig. 2.

Hence, the marginal effect of the flexible return policy between the manufacturer and the retailer is

\[ \text{ME}_{P-D} = EII_b^2(Q^*_b) - EII_D^2(Q^*_b) \]

\[ = (p + g - s) \cdot \text{AREA}_{ABC}, \]

where AREA_{ABC} is the total area of regions B and C in Fig. 2. ME_{P-D} can be further divide into two parts, IE_{P-D} and EE_{P-D}. IE_{P-D} is the internal effect of the contract which is the increased profit of the manufacturer and the retailer (who are inside the contract), and EE_{P-D} is the external effect of the contract which is the increased profit of the supplier (who is outside the contract).

\[ \text{EE}_{P-D} = IIP_b^2(Q^*_b) - IIP_D^2(Q^*_D) \]

\[ = (p + g - s) \cdot [F(Q^*_b) - F(Q^*_D)] \cdot (Q^*_b - Q^*_D) \]

\[ = (p + g - s) \cdot \text{AREA}_{ABC} \]

\[ \text{IE}_{P-D} = \text{ME}_{P-D} - \text{EE}_{P-D} \]

\[ = (p + g - s) \cdot \text{AREA}_{ABC} - (p + g - s) \cdot \text{AREA}_{AC} \]

\[ = (p + g - s) \cdot \text{AREA}_{AB}, \]

where AREA_{AB} and AREA_{AC} are the area of region B and C, respectively, in Fig. 2.

After the supplier and the manufacturer introduce a second return policy in scenario B based on scenario P, the expected channel profit becomes

\[ EII_b^2(Q^*_b) = (p + g - s) \cdot \int_0^{Q^*_b} x \, dF - g \cdot \mu \]

\[ = (p + g - s) \cdot \left[ F(Q^*_b) \cdot Q^*_b - \int_0^{Q^*_b} F(x) \, dx \right] - g \cdot \mu \]

\[ = (p + g - s) \cdot \text{AREA}_{ABCD} - g \cdot \mu, \]

where AREA_{ABCD} is the total area of regions A, B, C and D in Fig. 2. So, the marginal effect of the second return policy is

\[ \text{ME}_{B-P} = EII_b^2(Q^*_b) - EII_D^2(Q^*_b) \]

\[ = (p + g - s) \cdot \text{AREA}_{AD}, \]

where AREA_{AD} is the area of region D in Fig. 2.

The two return policies also benefit the upstream firms of the supplier as the order quantity increases, however such effects are not considered here as the scope of investigation is limited to the firms within the three level supply chain.

**Property 1.** The marginal effects of the contracts mainly rely on the increments of the order quantities, which are highly correlated with the marginal sales revenues of the contract-offering firms before introducing the contracts. With same increments of the order quantities, the marginal effect of the flexible return policy between downstream firms is apt to be larger than that of the return policy between the upstream firms.

### 4.5 Numerical studies and sensitive analysis

This section provides some numerical examples. Let \( c = 35, c^S = 5, c^M = 15, c^R = 5, s^S = 10, s^M = 5, s = s^S + s^M = 15, g = 5, p = 130. \) Two kinds of customer demands, normal and uniform distributions are considered, both with means 1000 and standard deviations 300. With each distribution, three groups, S, M and R, are investigated. In group S, the supplier has relative large marginal sales revenue than the other two firms, so do the manufacturer in group M and the retailer in group R. In each group, five different scenarios are studied.

Take the Group S with normal distribution as an example. At first, the transfer prices in scenario D are negotiated by the three firms, without loss of generalization, say \( w^S = 80 \) and \( w^M = 110. \) So the marginal sales revenues of the supplier is 40, larger than that of the manufacturer and of the retailer (both are 15). In scenario P1, we assume that the supplier and the manufacturer agree to the same transfer price, i.e., \( w^S \) is still 80. The manufacturer and the retailer implement a flexible return policy in which they agree to allocate their total profit with the same ratios between them in scenario D. In scenario B1, the supplier and the manufacturer introduce a second return policy based on scenario P1 and they agree that the supplier gets the same proportion of the total channel profit as in scenario P1. Scenarios P2 and B2 are very similar to P1 and B1 except that...
the supplier and the manufacturer agree to decrease $w^S$ from 80 to 72 (about 10%) for the increased order quantity. See Tables 1 and 2 for the optimal contracts prices, order quantities and expected profits with normal and uniform demand distributions.

Tables 1 and 2 provide experimental evidence to support the viewpoint that the flexible return policy between the manufacturer and the retailer has more conspicuous effect in group M where the manufacturer has relative large marginal sales revenues than in group S and R, and the return policy between the supplier and the manufacturer has more distinct effect in group S where the supplier has relative large marginal sales revenues than in group M and R. Moreover, the flexible return policy between the manufacturer and the retailer generally has larger effect than the return policy between the supplier and the manufacturer. Although the channel can be fully coordinated by two return policies, it is unnecessary to implement the contracts unless they can improve the performance remarkably as we have not considered the costs of implementing the contracts yet.

The impact of the demand distribution on the optimal prices also deserves attention. When the demand follows

### Table 1
Decisions and expected profits of the firms and of the channel with normal demand distribution

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### Table 2
Decisions and expected profits of the firms and of the channel with uniform demand distribution

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normal distribution and \( \mu \gg 3\sigma \),

\[
\frac{\mu}{\lambda} = \left\{ 1 - \frac{\sigma}{\mu} \cdot \frac{1}{\sqrt{2\pi} \cdot K} \cdot e^{-\Phi^{-1}(K)^2/2} \right\}^{-1},
\]

where \( \Phi \) is the cumulative distribution function of the standard normal distribution. When the demand follows uniform distribution,

\[
\frac{\mu}{\lambda} = \left\{ 1 - \frac{\sigma}{\mu} \cdot \sqrt{3} \cdot (1 - K) \right\}^{-1}.
\]

With both kinds of distributions, we have

\[
\sigma/\mu \uparrow \Rightarrow \mu/\lambda \uparrow \Rightarrow \rho, \delta^T \downarrow \\
\Rightarrow (w^M)^*, (b^M)^*, (w^S)^*, (b^S)^* \downarrow
\]

**Property 2.** When the whole channel is fully coordinated by appropriate return policies and the retailer faces normal or uniform demand, more capacious market will lead to higher internal prices while more volatile market will result in lower internal prices, if other parameters keep unchanged.

Consider two different demand distributions \( F \) and \( G \), let \( |\Delta(\mu/\lambda)| = |\mu_F/\lambda_F - \mu_G/\lambda_G| \), then the errors of the optimal contracts prices are

\[
\begin{align*}
|\left( w^{S}\right)^*_F - \left( w^{S}\right)^*_G | = & |\Delta(\mu/\lambda)| \cdot g \cdot \alpha^S \\
|\left( b^{S}\right)^*_F - \left( b^{S}\right)^*_G | = & |\Delta(\mu/\lambda)| \cdot g \cdot \alpha^S/K \\
|\left( w^{M}\right)^*_F - \left( w^{M}\right)^*_G | = & |\Delta(\mu/\lambda)| \cdot g \cdot (\alpha^S + \alpha^M) \\
|\left( b^{M}\right)^*_F - \left( b^{M}\right)^*_G | = & |\Delta(\mu/\lambda)| \cdot g \cdot (\alpha^S + \alpha^M)/K.
\end{align*}
\]

(22)

Note that \( \mu/\lambda \) is a very important indicator of the channel. According to (9), \( \mu/\lambda \) is always greater than 1. Table 3 lists \( \mu/\lambda \) with different critical ratios and different \( \sigma/\mu \) with normal and uniform demand distributions. When the market risk \( \sigma \) is smaller, the market volume is huger, or the ideal critical ratio \( K \) is larger, \( \mu/\lambda \) is smaller and approaches 1, which means the profit-capture ability of the channel is stronger.

Table 4 lists \( \mu/\lambda \) with ten kinds of different demand distributions which all have mean 1000 and standard deviation 300.

When the retailer misestimates the demand distribution, from (22) we know that the errors of the optimal contracts prices mainly depend on \( g, \alpha \) and \( |\Delta(\mu/\lambda)| \). When the goodwill cost can almost be ignored, i.e., \( g \rightarrow 0 \), the optimal contracts prices are almost independent of the demand distribution. When the goodwill cost is very important and can not be ignored, Table 3 and 4 indicate that:

1. \( |\Delta(\mu/\lambda)| \) decreases when \( K \) increases as shown in Table 3. For example, with normal distribution and \( \sigma/\mu \) shifting from 0.10 to 0.30, \( |\Delta(\mu/\lambda)| \approx 0.40 \) when \( K = 0.3 \) while \( |\Delta(\mu/\lambda)| \approx 0.17 \) when \( K = 0.6 \). So, more profitable channel has relative smaller prices errors;
2. \( |\Delta(\mu/\lambda)| \) increases when the start point of shifting of \( \sigma/\mu \) increases as shown in Table 3. For example, with normal distribution and \( K = 0.5 \), \( |\Delta(\mu/\lambda)| \approx 0.10 \) when \( \sigma/\mu \) shifts from 0.10 to 0.20 while \( |\Delta(\mu/\lambda)| \approx 0.12 \) when \( \sigma/\mu \) shifts from 0.20 to 0.30. So, capacious market has relative small
prices errors while volatile market has relative large prices errors; and

$3. \vert \Delta(\mu/\lambda) \vert$ is rather small between kinds of different distributions with same mean and same standard deviation as shown in Table 4. For example, when $\mu = 1000, \sigma = 300, \vert \Delta(\mu/\lambda) \vert \approx 0.05$ between normal and gamma distributions when $K = 0.3, \vert \Delta(\mu/\lambda) \vert \approx 0.02$ between uniform and Weibull distributions when $K = 0.6$, etc.

Property 3. The errors of the optimal contracts prices are not very sensitive to the demand distributions. More profitable channel and more capacious market lead to relatively smaller prices errors while more volatile market results in relative larger prices errors.

5. Conclusion

This paper studies the coordination issue of a three level supply chain selling short life cycle products in a single period model. The contract between the downstream firms is negotiated before that between the upstream firms. If the manufacturer can not precisely anticipate the contract prices the supplier is willing to offer, full coordination can only be achieved by little chance. We construct the so-called flexible return policy by just setting the rules of pricing while postponing the determination of the final contract prices. A flexible return policy generally consists of two terms to set the contracts prices. A critical ratio term to determine the total profits of related firms, and a profit-sharing term to allocate the profit properly among the firms. With return policies between each pair of adjacent firms, unsold products or used modules disassembled from the unsold products are returned level by level from the retailer to the upstream firms and each firm shares in the loss due to the overstock. We show that the three level supply chain can be fully coordinated by appropriate contracts and the total profit of the channel can be allocated with any specified ratios among the firms.

With numerical studies, we indicate that the optimal prices in the coordinated system are not very sensitive to the demand distributions. Imprecise estimation of the mean and standard deviation of the demand, and even inexactitude judgment of the demand distribution type only leads to little errors, which is a favorable characteristic to the decision makers.

The marginal effects of the contracts mainly rely on the increments of the order quantities, which are highly correlated with the marginal sales revenues of firms. Furthermore, when with same increments of the order quantities, the marginal effect of the flexible return policy between the manufacturer and the retailer is apt to be larger. Multi-level supply chain can be fully coordinated if each pair of adjacent firms implement flexible return policies. However, it is unnecessary to implement every contract unless it can improve the performance remarkably.

More research works such as considering the cooperation of different kinds of contracts in complex supply chains will be carried on in the future.

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References