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Hierarchical coordination mechanisms within the supply chain

Christoph Schneeweiss *, Kirstin Zimmer

Chair of Operations Research, University of Mannheim, P.O. Box 10 32 64, D-68131 Mannheim, Germany

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Abstract

The paper analyses operational coordination mechanisms between a producer and a supplier within a supply chain having private local information. For a make to order production setting the coordination is achieved through the combined use of a task-oriented and a control-oriented type of instrument. The task-oriented instrument describes the producer’s procurement policy for components whereas the control-oriented instrument is made up by penalty costs for non-correct delivery. In using both of these instruments various coordination schemes are employed in making use of the theory of hierarchical planning. Though it is assumed that producer and supplier possess some private information they are not taken to behave antagonistically. The question, which information should be disclosed, is of central importance for the overall performance of the supply chain and is the main focus of an extensive quantitative analysis which finally is used to give recommendations for the design of a supply contract.

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1. Introduction

In recent years, supply chain management (SCM) has undergone a rapid development in theory and practice. Growing globalisation together with an increased focus on their core competence has led companies to split their logistic processes into various fairly independent units. These so-called decision making units (DMUs) can be factories or haulage contractors within a network of material and information flows. SCM may therefore be considered as a management activity that has to do with the coordination of logistic processes being locally controlled by various independent DMUs. Thus the term ‘SCM’ is used here as a more operational activity within an existing supply network. Strategic design questions are not considered to be part of SCM.

Both attributes, ‘logistic’ and ‘independent’, are of importance for SCM to be a theoretically new and exciting field: For totally dependent DMUs, ordinary logistic decision procedures would apply, as they are well-known within a single company. On the other hand, if ‘logistic’ were not of
importance but rather the term ‘independent DMUs’, one would end up with micro-economic considerations, possibly with well-established theories like contracting, auction theory, or principal agent theory without any explicit relationship to operational decisions. Hence, from a theoretical point of view, it is the existence of a detailed (logistic) decision space together with private information and independent decision criteria which makes SCM a challenging field of logistics and operations management. Or, to put it differently, it is the paradigmatic shift from central to distributed decision making that calls for new efforts in logistics and production management.

In much of the existing literature on (operational, non-strategic aspects of) SCM, the extension of logistics to more than one independent decision maker has not really been made. Many logistics-based approaches to SCM are still within the traditional realm of one central DMU. The same holds just as true for most of the existing supply chain software.

On the other extreme, one describes the supply chain on a highly aggregated level as a network of antagonistically behaving DMUs each of them possessing private information. Taking explicitly into account only a few logistic constraints, the focus is on contracting as it is analysed in micro-economic theory. Usually, these contracts are concerned with fixing purchasing prices and amounts of material to be shipped within a certain span of time. No detailed operational procedures are considered to be an objective of these contracts.

The investigation we are going to present does not have primarily to do with contracting but rather with operational coordination mechanisms within the supply chain. These coordination mechanisms, however, could be part of a supply chain contract. More precisely, we are going to investigate several operational coordination mechanisms of increasing complexity between the supplier and the producer both possessing some private information. In particular, we are analyzing the effect of a possible disclosure of private information on the overall performance of the supply chain allowing us to give recommendations for the design of a supply contract.

The investigation is organized as follows. After a brief review of related literature we are describing in Section 3 the type of supply chain we are investigating (Section 3.1) and define the specific decision model we are going to analyse (Section 3.2). Since the relationship between supplier and producer may conveniently be described within the framework of hierarchical planning, we are presenting some essential concepts of this theory in Section 4.1. Section 4.2 is then deriving key coordination mechanisms which, in Section 5, are exposed to an extensive numerical investigation. Finally, Section 6 provides a summary and some additional observations as to the interpretation of the considered coordination schemes.

2. Related literature

As mentioned in the introduction the literature on SCM can be roughly split into two classes. The first class describes the supply chain on a highly aggregated level as a network of antagonistically behaving DMUs each of them possessing private information. Since these approaches take explicitly into account coordination problems of decentralized decision making let us call this class coordination-oriented SCM. The second class, more logistically based, considers the supply chain from the point of view of one central decision maker minimizing overall total cost.

(1) Coordination-oriented SCM may be classified into the three categories: (i) quantity discounts, (ii) inventory control and (iii) contracting. Monahan (1984) is the first to consider the effect of quantity discounts in a two-echelon system consisting of one supplier and one buyer. He investigates the coordination problem from the supplier’s point of view and shows that the supplier is able to increase his profit by offering quantity discounts. These results induce a series of extensions (e.g., see Banerjee, 1986; Lee and Rosenblatt, 1986; Joglekar, 1988; Weng, 1995). Corbett and de Groote (2000) show that under the full information assumption these schemes are all equivalent and derive the optimal quantity discount under asymmetric information. For an overview see Gojal and

In the context of inventory control, incentive coordination issues are analysed by Porteus and Whang (1991), Lee and Whang (1999) and Chen (1999). They consider supply chains consisting of divisions of the same firm each of them having private information. Porteus and Whang investigate the coordination of a firm consisting of different marketing managers and one manufacturing manager under asymmetric information using a multi-product newsvendor model. They derive an optimal incentive plan that induces the managers to act so that the expected returns to the owners of the firm are maximized. Lee and Whang determine a performance measurement scheme for a decentralized version of the model of Clark and Scarf (1960). Chen extends the work of Lee and Whang and analyses the impact of information delays and non-rational behavior. While the incentive scheme of Lee and Whang relies on transfer payments between the different divisions, Chen considers transactions between the owner of the firm and the divisions.

The interest in the field of supply chain contracting has grown in recent years. Taking explicitly into account only a few logistic constraints, the focus is on analyzing the impact of contract parameters on the overall performance of the supply chain. For an overview see Tsay et al. (1999) and Cachon (1999). While in the context of inventory control the decision makers are usually divisions belonging to the same company, the contrary in the case for supply chain contracting. Frequently investigated contract parameters are quantity commitments (Anupindi and Bassok, 1999), returns policies (Lariviere, 1999), and options (Barnes-Schuster et al., 1998).

(2) Logistics-oriented models in SCM vary in their focus on operational details, like material flows or production planning. In most of the existing literature the transition to more than one independent decision maker has not really been made. First considerations in this direction may be found in Geoffrion and Graves (1974), who introduce a multi-commodity logistics network model for optimizing product flows. Cohen and Moon (1991) present a mixed integer multicommodity model to find inbound supply and outbound distribution flows. They develop an algorithm to solve a production-distribution model for the case of a piecewise linear concave cost function. Arntzen et al. (1995) derive a multi-period, multi-commodity mixed integer model in order to optimize the global supply chain at the Digital Equipment Corporation. In contrast to the coordination-oriented investigations all these approaches view the supply chain as a fully vertically integrated firm where all information is common knowledge and the material flow is controlled by a single decision maker (see also Martin et al., 1993; Geoffrion and Powers, 1995; Vidal and Goetschalckx, 1997; DeCroix and Arreola-Risa, 1998). The same holds just as true for most of the existing supply chain software (e.g., see SAP-APO, 2000 or RHYTHM, 1999 of i2).

For the analysis we are going to present, the most interesting study in the field of logistics-oriented SCM is the work of Kolisch (2000), who is the first to consider the integration of heterogeneous production systems, i.e., the integration of in-house assembly and manufacturing. He presents a MIP-formulation of this problem and develops an efficient solution heuristic. In contrast to Kolisch we focus on the coordination of in-house assembly and out-house manufacturing. This implies that problems of distributed decision making are to be discussed and particularly those having private information. Since up to now private information is only discussed in incentive-oriented coordinations of the supply chain we are thus combining both main directions in SCM the coordination-oriented direction and the logistics-oriented track. In particular, we are using a (centralized) model related to that of Kolisch as a benchmark to evaluate our non-centralized (team-oriented) coordination mechanisms.

3. Problem statement

3.1. General framework

Let us focus on a supply link within a supply network consisting of a producer and a supplier. As depicted in Fig. 1, to meet (external) customer
demand (of the consumer market) the producer places orders for some necessary components with a supplier, which, contingent on the supplier’s capacity situation, are more or less correctly carried out. More precisely: the delivery of components might not match the ordered amount and might not be in time. However, for a longer horizon, say 10 weeks, the total amount is fixed according to a general procurement contract and is delivered correctly.

Hence, we are considering a hierarchical relationship between the producer and the supplier with the producer being the upper level and the supplier representing the lower level. Taking the position of the producer, we particularly assume that she is not fully informed of the supplier’s decision situation, especially of the operational capacity conditions. To coordinate the link, we establish as a typical control measure a penalty cost for not delivering the correct amount in due time. That is, for given total external demand, we are considering an (operational) short-term coordination of ordering and adjoint delivery decisions over a total horizon of \( t = 1, \ldots, T \) periods (see Fig. 2). Consequently, an order \( q \) represents a whole sequence \( q = (q_1, \ldots, q_T) \) of components orders, and the subsequent delivery, \( d = (d_1, \ldots, d_T) \), tries to match such an order, possibly deviating from \( q \) by an amount of \( \delta \).

To put it in more general terms, the (top-down) influence consists of a ‘task-oriented’ instruction \( (q) \) and a ‘control-oriented’ instruction \( (K) \). This latter instruction could be interpreted as a ‘leadership activity’ and is exclusively employed to guarantee the correct execution.

Taking a closer look at the state of information, we reasonably assume that both parties maintain some privacy, i.e., they do not reveal all their data, which consequently results in an asymmetric information state. This situation, however, does not give rise to an opportunistic behavior and thus does not imply a principal agent type of situation. Assuming an opportunistic behavior, one would not capture essential features of a supply chain which is usually intended to establish a long-term trusting cooperation. Hence, we presume a team-like behavior, i.e., all parties attempt to optimize a common goal. Still there exists private information, and it is one of our main concerns to quantify the effect of private information on the overall performance of the supply link.

Usually a supply link has to be embedded into the entire supply network. As an example, the supplier might not have just one but several other producers he is in contact with (Fig. 2). In capturing this influence of the full network, we describe the availability of the supplier’s capacity as being stochastic. More precisely, at the time when the producer places her purchasing order \( q \) with the supplier, the capacity is only stochastically known for both parties. Only later, when the delivery decision is to be made, the supplier is assumed to know all his commitments, and hence the capacity that is still available. This externally induced uncertainty (for the producer) adds to the lack of knowledge she already has with the (partially) unknown characteristics of the supplier.

Having described the principle structure of the supply link we may now state the primary focus of our analysis in more operational terms. In view of the two kinds of top-down influence, the task-oriented and the control-oriented instruction, two questions are of primary interest:

1. Which is the combined effect of a task-oriented and a control-oriented instruction?
(2) Which is the effect of information asymmetry between producer and supplier on the performance of the supply link?

To investigate these questions let us define more closely the decision models which will be used as a basis for the subsequent quantitative analysis.

3.2. Formal description of the supply link

Let us first describe the producer’s and the supplier’s model separately and let us show how these models are to be linked. Later, in Section 4, we then explain how the coordination of the two levels achieved by the three influences depicted in Fig. 2 can be optimized.

3.2.1. The producer’s model (P)

Having in mind the particular type of control-oriented influence (due date penalty cost), it seems to be adequate to describe a make to order production situation (e.g., see Kolisch, 2000). More precisely, the company produces several products which are ordered by some (external) customers. Thus the producer’s model will be taken to be a capacitated project planning model. To hold the quantitative analysis simple, let us assume for the entire planning horizon that customer orders are to be known. These orders are representing a sequence of manufacturing jobs which are assigned the same (external) due date. To fulfill the (external) customer orders, the producer has to place orders with her supplier to procure her with the necessary components.

Objective function

\[ C^T = \sum_{t=1}^{T} \left( cY_t + c^+Y_t^+ + c^-Y_t^- \right) \]

\[ + \sum_{t=1}^{T} \sum_{i=1}^{I} \left( p_i \delta^d_i - K \delta^h_i - c \delta^h_i \right) + \sum_{i=1}^{I} c_y_i \]

\[ + \sum_{t=1}^{T} \sum_{i=1}^{I} h_i^p I_i + \sum_{i=1}^{I} F \max_{j \in J_i} \Delta_j \]

\[ \rightarrow \min. \]  

Constraints

Capacity adaptation constraints:

\[ Y_{t+1} = Y_t + Y_t^+ - Y_t^- \quad \forall t = 1, \ldots, T - 1, \quad (P2) \]

\[ Y_1 = Y_1', \quad (P3) \]

\[ Y_t^+ \leq Y_{t+1}^+ \quad Y_t^- \leq Y_{t+1}^- \quad \forall t. \quad (P4) \]

Capacity constraints:

\[ \sum_{j=1}^{J} \sum_{k=1}^{K} a_{jk+1} x_{jk} \leq Y_t + Y_t' \quad \forall t = 1, \ldots, T. \quad (P5) \]

Network constraints:

\[ \sum_{t=1}^{T} x_{jt} = 1 \quad \forall j, \quad (P6) \]

\[ \sum_{t=1}^{T} (t + D_j) x_{jt} \leq E_t + A_j \quad \forall j \in J_t, \forall l. \quad (P7) \]

\[ \sum_{t=1}^{T} (t + D_h) x_{ht} \leq \sum_{t=1}^{T} t x_{ht} \quad \forall h, j \in P(j). \quad (P8) \]

Material balance constraints:

\[ \sum_{j=1}^{J} \nu_{ji} x_{ji} \leq \delta^{d0}_i + I_{il} \quad \forall i, t, \quad (P9) \]

\[ I_{i,t+1} = I_{il} + \delta^{d0}_i - \sum_{j=1}^{J} \nu_{ji} x_{ji} \quad \forall i, t, \quad (P10) \]

\[ I_{il} = 0 \quad \forall i. \quad (P11) \]

Order delivery constraints:

\[ \delta^{d0}_i + \delta^{h0}_i - \delta^{h-}_i \quad \forall i, t, \quad (P12) \]

\[ \delta^{d0}_i + \delta^{h0}_i - \delta^{h-}_i = q_{il} \quad \forall i, \quad (P13) \]

\[ \delta^{d0}_i = \text{AF}(q_{il}) \quad \forall i, t. \quad (P14) \]

Integrality and non-negativity constraints:

\[ Y_t, Y_t^+, Y_t^- \geq 0. \quad (P15) \]

\[ q_{il}, \delta^{d0}_i, \delta^{h0}_i, \delta^{h-}_i, Y_t, I_{il}, A_j \geq 0 \quad \forall i, j, t. \quad (P16) \]

\[ x_{jt} \in \{0, 1\} \quad \forall j, t. \]
The objective function $C^T$ of the producer describes in its first component capacity costs and capacity adaptation costs. The term $\sum_{t=1}^T \sum_{j=1}^J p_dj_t$ represents purchasing costs, and $\sum_{t=1}^T \sum_{j=1}^J K\delta_{jt}$ penalty cost, i.e. the supplier’s payment to the producer, in a given period $t$, for delayed delivery (see Eq. (P12)). Similarly, the expression $\sum_{t=1}^T \sum_{j=1}^J \epsilon j_{it}$ describes payments for premature deliveries. The term $\sum_{t=1}^T \sum_{j=1}^J h^P_{it}$ represents holding costs for components being stocked at the producer and $\sum_{t=1}^T F \max_{j\in J} A_j$ stands for costs the producer is charged for not serving her (external) customers in due time. Finally $\sum_{t=1}^T \sum_{j=1}^J \epsilon j_{it}$ gives (the producer’s) over time cost in order to reduce stockouts. Summarizing, the first sum describes overall capacity costs, the next two terms represent the direct financial relationship with the supplier, and the last three terms describe the consequences in case the supplier does not deliver in time. Note that the last term is related more explicitly to the (external) customers.

The constraints consist of five categories:

The capacity adaptation is described by the balance equation (P2), with the initial capacity amount given in Eq. (P3) and adaptation constraints (P4). Expression (P5) represents, as usual, the capacity constraint. Note that a job in time $t$ which started in time $k$ has been processed for $t + 1 - k$ periods, and hence has a capacity consumption of $d_{jt+1-k}$ units. Eq. (P6) makes sure that each job $j$ starts only once. The due date is guaranteed by relation (P7), and the temporal network structure is described by (P8).

The material balance constraints are given by relations (P9) through (P13). Constraint (P9) makes sure that a job $j$ in time $t$ can only be produced if the necessary components were made available by the supplier. Eq. (P10) defines, as usual, the material balance equation with initial condition (P11). Eqs. (P12) and (P13) describe positive and negative deviations ($\delta^+, \delta^-)$ between ordered $(q)$ and delivered $(d)$ quantities. They represent the order–delivery relationship. It is these deviations that are controlled by the penalty costs.

The most interesting relation is given by (P14). It defines the delivery $d_{jt}$ as the anticipated delivery decision of the supplier. In general, this decision depends on the order quantity $q_{it}$ and has to be obtained by the producer from certain anticipated
features of the supplier’s production model described by the anticipation function \( AF(q_{it}) \). The next section will describe, in some more detail, the significance of AF. First, however, let us look at the situation of the supplier.

3.2.2. The supplier’s model (S)

The supplier’s model is assumed to be a linear (mixed integer) lotsizing model considering setup costs but no setup times.

**Objective function**

\[
C^B = \sum_{i=1}^{T} \sum_{t=1}^{T} \{ p_d i_{it} - h^S_{it} i_{it} - s_i z_{it} - K \delta^+_{it} - \epsilon \delta^-_{it} \} \\
- \sum_{i=1}^{T} \kappa \Delta C_t \rightarrow \text{max.} \quad (S1)
\]

**Constraints**

Material balance constraints:

\[
i_{i,t+1} = i^0_{it} + Q_{it} - d_{it} \quad \forall i, t, \quad (S2)
\]

\[
i_{it}^0 = l_{it} \quad \forall i, t, \quad (S3)
\]

\[
Q_{it} \leq M z_{it} \quad \forall i, t. \quad (S4)
\]

Capacity constraints:

\[
\sum_{i=1}^{T} c_i Q_{it} \leq z_{it} C + \Delta C_t \quad \forall t, \quad (S5)
\]

\[
\Delta C_t \leq \Delta C_t^{\text{max}} \quad \forall t. \quad (S6)
\]

Order–delivery relationships:

\[
d_{i,t+1} + \delta^+_{i,t+1} - \delta^-_{i,t+1} = q_{i,t+1} + \delta^+_{i} - \delta^-_{i} \quad \forall i, t, \quad (S7)
\]

\[
d_{it} + \delta^+_{it} - \delta^-_{it} = q_{it} \quad \forall i, \quad (S8)
\]

\[
\sum_{i=1}^{T} d_{it} = \sum_{i=1}^{T} q_{it} \quad \forall i. \quad (S9)
\]

Integrality and non-negativity constraints:

\[
z_{it} \in \{0, 1\} \quad \forall i, t, \quad (S10)
\]

\[
Q_{it}, d_{it}, \delta^+_{it}, \delta^-_{it}, h^S_{it}, r^S_{it}, \Delta C_t \geq 0 \quad \forall i, t. \quad (S11)
\]

**Decision variables**

\[
d_{it} \quad \text{quantity of component } i \text{ delivered in period } t
\]

\[
i^0_{it} \quad \text{inventory of component } i \text{ in period } t \text{ (supplier)}
\]

\[
z_{it} \quad \text{setup indicator} \quad (z_{it} = 1, \text{if component } i \text{ is manufactured in period } t)
\]

\[
\delta^+_{it} \quad \text{number of units of component } i \text{ which are ordered but not delivered at time } t
\]

\[
\delta^-_{it} \quad \text{number of units of component } i \text{ which are delivered but not ordered at time } t
\]

\[
\Delta C_t \quad \text{increase in capacity in period } t
\]

\[
Q_{it} \quad \text{production amount of product } i \text{ in period } t
\]

**Constants**

\[
h^S_i \quad \text{holding cost coefficient of component } i \text{ (supplier)}
\]

\[
s_i \quad \text{setup cost of component } i
\]

\[
\kappa \quad \text{cost to increase the capacity by one unit}
\]

\[
l_{it}^0 \quad \text{initial inventory of component } i
\]

\[
q_{it} \quad \text{order quantity of component } i \text{ in period } t
\]

\[
c_i \quad \text{manufacturing coefficient of component } i
\]

\[
C \quad \text{normal capacity}
\]

\[
z_{it} C \quad \text{capacity available in period } t
\]

\[
\Delta C_t^{\text{max}} \quad \text{maximal increase in capacity}
\]

The objective function (S1) represents the contribution margin of the supplier. Besides the positive revenue term \( \sum_{i=1}^{T} \sum_{t=1}^{T} p d_{it} \), one has inventory, setup, and penalty costs, respectively. \( K \delta^+_{it} \) accounts for the producer’s penalty for not keeping the due date while \( \epsilon \delta^-_{it} \) describes costs incurred because of a premature delivery. The last term stands for costs caused by a capacity extension \( \Delta C_t \).

The constraints (S2) and (S3) describe the usual inventory constraints, whereas (S4) links the production variable \( Q_{it} \) to the setup variable \( z_{it} \). Eqs. (S7) and (S8) represent the relationship between order and delivery, and (S9) takes account of the (binding) contract that total ordered material is ultimately to be delivered. The most interesting constraint is (S5). With \( z_{it} \in [0, 1] \), it says that (normal) capacity usually will not be fully available. As discussed in the previous section, in contrast to the supplier, \( z_{it} \) is not known to the
producer and can be considered as part of the supplier's private information.

There are several actions available to the supplier to react to the ordering sequence \((q_1, \ldots, q_T)\) and the particular value of \(a_t\). He can expand his capacity \((\Delta C_t)\), build up inventories, or he can accept penalties and, of course, he can employ a mixture of all these instruments.

Summarizing, for the setting to be investigated, one has the following sequence of decisions (see Fig. 3). Prior to the planning horizon \([1, \ldots, T]\), the producer decides in \(\tau_0\) on her ordering vector \((q_1, \ldots, q_T)\) and penalty cost parameter \(K\). In doing so, she anticipates the deliveries \((\hat{d}_1, \ldots, \hat{d}_T)\) of the supplier. Later, in time \(\tau_1\), but still prior to the first period of the planning horizon, the supplier chooses, in view of his known available capacity (given by revealed values \(a = (a_1, \ldots, a_T)\)) actual deliveries \((d_1, \ldots, d_T)\) of which, at least \(d_1\), is actually executed. The remaining ones could possibly be recalculated in a rolling horizon fashion as new information enters the system. The present investigation, though, will not discuss such an extension of the stated problem. Note, however, demand is considered as being deterministic. This allows us to focus exclusively on the uncertainty w.r.t. important characteristics of the supplier. Obviously, for the link as a whole, penalty costs cancel out each other. Yet, they have an important effect. They inform the supplier of the cost consequences for the producer of his untimely deliveries.

Clearly, the relation between the two models is achieved by the instruments depicted in Fig. 3. The bottom-up influence is effecting the producer via Eq. (P14), and the top-down instruction influences the supplier via Eqs. (S1) and (S8). To employ these instruments most effectively we particularly need an optimal expression of the anticipation function \(AF(\cdot)\) in Eq. (P14) which will be discussed in the subsequent section.

4. Basic concepts of hierarchical planning and types of anticipation and coordination

In coordinating the supply link using the provided instruments (as shown in Fig. 3) one may choose several approaches. They all differ in the calculation of AF. Thus, before investigating different types of coordination, in Section 4.2, let us explain the concept of anticipation within the theory of hierarchical planning.

4.1. Basic concepts of hierarchical planning

In terms of hierarchical planning the producer takes the part of the top-level whereas the supplier is representing the base-level. Let us denote the model of the top-level by \((C^T, A^T)\) with top-criterion \(C^T\) and top-decision space \(A^T\) and decision \(a^T \in A^T\). Similarly, one has, for the base-level (supplier), \(C^B\) and \(A^B\) and \(a^B \in A^B\). Moreover, let us introduce the information states \(I^T\) and \(I^B\) comprising the random influences of the top and base-level, respectively. The two levels are connected by a top-down and a bottom-up influence (see Fig. 4).

The **top-down influence** is called instruction \(IN\) which is a function of the top-decision \(IN = IN(a^T)\). This instruction can, in principle, influence all the components of the base-model, i.e., \(C^B\).
$A_{IN}^R, A_{IN}^L$. For the model of the previous section, one has $IN = \{(q_1, \ldots, q_T), K\}$.

The bottom-up influence is more complicated. It is, in principle, the anticipation of the base-level behavior which has to be taken into account by the top-level. If this anticipation does not depend on the instruction, it is called non-reactive. In the reactive case, however, the anticipation depends on IN and gives rise to the anticipation function $AF = AF(IN)$. The anticipation function describes the possible optimal reaction of the base-level. It can be viewed as an optimal anticipated (not necessarily realized) response to an impulse IN. In principle, it can influence criteria and decision space of the top-model: $C_{AF}^T$ and $A_{AF}^T$.

There are numerous ways to determine $AF(IN)$. A rational procedure is to estimate the components of the base-model $\hat{C}_{IN}, \hat{A}_{IN}$ and to calculate $AF(IN)$ as the optimal (estimated) base-level decision $AF(IN) = \hat{a}^{B}(IN)$. Hence, one obtains the following coupling equations:

$$a^T = \arg \max_{a^T \in A_{AF}^T} E\{C_{AF}^T | I^T\},$$  

(1a)\[ \text{IN} = \text{IN}(a^T), \]

$$AF(IN) = \arg \max_{a^B \in A_{IN}^B} E\{\hat{C}_{IN}^B(a^B) | I^B\},$$  

(1b)\[ a^B = \arg \max_{a^B \in A_{IN}^B} E\{C_{IN}^B(a^B) | I^B\} \]

(For a more detailed description, see Schneeweiss (1999).) These coupling equations and particularly (1b) will now be employed to calculate various approximations of $AF$.

4.2. Types of anticipation and coordination

Let us consider four models of the supply link which describe an increasing intensity of integration of the two models (P) and (S). On the one extreme, the producer (P) is assumed not to take into account any feature of the supplier (S) (pure top-down hierarchy), whereas on the other extreme the supplier is fully integrated into the producer (P) such that an anticipation disappears (ideal link).

4.2.1. Pure top-down hierarchy

For the pure top-down hierarchy, the supplier’s model (S) is not taken into account. The top-level postulates that the delivered amount $d_t$ is identical with the ordered amount. Hence,

$$AF(q_{it}) = q_{it} \quad \forall i, t.$$  

(2)

Note that in this case $AF(q_{it})$ is not representing an anticipation function since no possible reaction to the producer’s instruction is taken into account. Consequently, because of Eq. (2), Eqs. (P12) and (P13) vanish, and the objective function is reduced to its so-called ‘private’ term $C^T$,

$$C^T = C^{TT} := \sum_{t=1}^{T} (cY_t + c^+Y^+_t + c^-Y^-_t).$$  

(3)

This reduction is possible since we reasonably assume that, in view of the known (external) customer demand, normal capacity is cheaper than overtime and all the other costs described in the last three terms of (P1). Hence, these terms need not be considered. In addition, the producer model (P) is, of course, not affected by a bottom-up influence, so that the terms $K\delta_t$ and $\epsilon\delta_t$ in (P1) are obsolete as well. Note that the expression $\sum_{t=1}^{T} \sum_{i=1}^{I} p_id_t$ is constant, and hence does not effect the optimization.

4.2.2. Non-reactive anticipation

In contrast to the pure top-down hierarchy, the non-reactive anticipation accounts for important features of the supplier’s model. A reaction (to the instruction), however, is again not taken into account. Hence, the anticipation function is becoming obsolete:

$$AF(q_{it}) = q_{it} \quad \forall i, t,$$  

(4)

i.e., $AF(q_{it})$ has again not the meaning of an anticipation function. Moreover, the constraints (P12) and (P13) can be omitted.

The supplier’s model is non-reactively anticipated by the set of relations:

$$\sum_{i=1}^{I} \hat{c}_i \hat{d}_t \leq \hat{a}_t \hat{C} + \hat{\Delta C}_t \quad \forall t,$$  

(5)

$$\sum_{i=1}^{I} \kappa \hat{\Delta C}_t \leq \hat{AL},$$  

(6)
with the ^ indicating estimations of the producer. Hence, the supplier is primarily taken into account by his presumed capacity (Eq. (5)) and by his aspiration level AL up to which he is willing to build up extra capacity. Hence, the supplier is essentially represented by his capacity situation which seems not be an unreasonable assumption.

4.2.3. Reactive anticipation

For the reactive anticipation, the anticipation function is a non-trivial part of the producer’s top-down criterion and her decision space. This function

\[ \Delta C_t \geq 0, \]

is obtained by the producer in anticipating and optimizing the entire base-level, i.e., the model (S). The anticipated model (S) is simply obtained by the producer in replacing unknown parameters by their estimates. For the numerical analysis of Section 5, this turns out to be a substantial link in. As we will see in the numerical analysis of Section 5, this turns out to be a substantial

penalty costs reflect the temporary situation the supply link is in. As we will see in the numerical analysis of Section 5, this turns out to be a substantial advantage as compared with the traditional determination of penalty costs in practice.

Penalties for premature deliveries are not our main concern and will not be optimized as it is done for delays. This is due to the fact that premature deliveries are only causing additional inventories for the producer, whereas delayed deliveries cause extremely unwanted stockouts and need therefore to be optimized. Similarly, purchasing costs, e.g., are also taken to be given but their possible optimization will not be our concern.

4.2.4. Ideal model

For the ideal situation, which will be used as a benchmark, producer and supplier are considered as one decision maker, i.e., they build a team without any information asymmetry. Consequently, no estimates are necessary and, in particular, the available capacity \( z_t C \) is known. The criterion of the unified model now describes total cost for both parties

\[ C = C^T + C^B \]

\[ = \sum_{t=1}^{T} (cY_t + c^+ Y_t^+ + c^- Y_t^-) \]

\[ + \sum_{t=1}^{T} c\zeta_t + \sum_{t=1}^{T} \sum_{i=1}^{I} h_i^p I_{it} + \sum_{t=1}^{T} F \max_{j \in J_t} A_j \]

\[ + \sum_{t=1}^{T} \sum_{i=1}^{I} (h_i^s s_{it} + s_{it}) + \sum_{t=1}^{T} \kappa \Delta C_t \to \min. \]

(ID1)

Since no coupling via the anticipation function is necessary, the Eqs. (P12)–(P14) become obsolete, and, of course, the delivered amount \( d_t \) is no longer anticipated but explicitly optimized in the modified model \( P \), hence \( \hat{d}_t = d_t \). Moreover, since the supplier’s model is now represented explicitly in \( P \), the following relations have to be added to the decision space of \( P \):

\[ l_{i,t+1}^s = l_{i,t}^s + Q_t - d_t \quad \forall i, t, \]

(ID2)

\[ l_{i,t}^s = l_{i,t}^s \quad \forall i, \]

(ID3)

\[ z_t M \geq Q_t \quad \forall it, \]

(ID4)

\[ \sum_{i=1}^{I} c_i Q_t \leq z_t C + \Delta C_t \quad \forall t, \]

(ID5)

\[ \Delta C_t \leq \Delta C_t^{\max} \quad \forall t, \]

(ID6)

\[ z_t \in \{0; 1\} \quad \forall i, t, \]

(ID7)

\[ d_t, l_{i,t}^s, Q_t, \Delta C_t \geq 0 \quad \forall i, t. \]

(ID8)

Summarizing, we have the following situation: The pure top-down case describes a setting in which the models are coupled by the three influences depicted in Fig. 2. However, there is no planning activity that exploits a possible knowledge of each other. One has the typical case of a merely transactional relationship which, up to now, has been rather common in SCM in practice.
and for which a fixed value of the penalty cost parameter $K$ is characteristic.

For the non-reactive anticipation, one has at least the effect that the producer makes use of some knowledge she has about the supplier which allows her to adapt her instruction ($q$) to her own and her supplier’s needs. Finally, for the reactive anticipation, the adaptation can even be further improved, since in this setting also the supplier’s actions can be controlled so that the common contribution margin can be increased. In this situation, $K$ represents an additional decision variable. $K$ is optimized such that the solution of the reactively anticipated model results in the lowest costs w.r.t. the ideal model.

5. Numerical analysis

As stated at the end of Section 3.1 and in view of the different types of coordination, the numerical analysis is now to clarify two questions:

(1) Which is, in view of the various anticipative coordination schemes, the combined impact of the task-oriented and the control-oriented type of instruction?

(2) Which is the effect of the different kinds of private information on the performance of the supply link?

In order to analyse these questions, the defined models (P) and (S) must be specified, and the supply link has to be solved for the different types of coordination as discussed in the previous section. For specified (external) customer demand, we still have the stochastics induced by the uncertainty w.r.t. the supplier’s capacity expressed by $z_t$. Thus in comparing the different coordination schemes, a risk analysis is necessary.

5.1. Specification of the analysis

Let us now specify models (P) and (S) as follows.

Model (P) is a capacity model (e.g., man power) with a work load given by a sequence of $l = 40$ orders comprising altogether $j = 1, \ldots, 100$ jobs. The planning horizon is $T = 10$ periods. The structure of the orders and the remaining data which define criteria and decision space are summarized in Fig. 5 and Tables 1–3.

Model (S) is fixed by the data of Table 4, and the specification of the random variables $z_t$ may be found in Table 5.

<table>
<thead>
<tr>
<th>Order type 1 (10 orders)</th>
<th>Job 1</th>
<th>Job 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order type 2 (20 orders)</td>
<td>Job 3</td>
<td>Job 4</td>
</tr>
<tr>
<td>Order type 3 (10 orders)</td>
<td>Job 6</td>
<td>Job 7</td>
</tr>
</tbody>
</table>

Fig. 5. Structure of external orders.

Table 1
Data of external orders

<table>
<thead>
<tr>
<th>Order type</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of orders</td>
<td>10</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Job</td>
<td>j</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Predecessor</td>
<td>$P(j)$</td>
<td>–</td>
<td>1</td>
</tr>
<tr>
<td>Number of comp. 1</td>
<td>$v_{j1}$</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Number of comp. 2</td>
<td>$v_{j2}$</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Duration</td>
<td>$D_j$</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Due date</td>
<td>$E_j$</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 2
Capacity consumption $a_{jt}$ of job $j$ in period $t$ after its start

<table>
<thead>
<tr>
<th>$a_{jt}$</th>
<th>Job $j$</th>
<th>period $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>
The supply link is specified such that penalties actually become effective for the chosen sequence of orders. This implies that the supplier, at least temporarily, has a bottleneck situation. Moreover, none of the instruments of the link should be unnecessary, i.e., the instruments of stocking, producing in advance, and capacity expansion should actually be employed. Finally, in choosing a high value for penalty cost $F$ for exceeding the due date we make sure that delayed deliveries are really an issue and hence, coordination mechanisms are indeed effective.

### Table 3
Data of model P

<table>
<thead>
<tr>
<th>Constants</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>0.5 ($/h)</td>
</tr>
<tr>
<td>$c^+$</td>
<td>1 ($/h)</td>
</tr>
<tr>
<td>$c^-$</td>
<td>0.2 ($/h)</td>
</tr>
<tr>
<td>$Y^+$</td>
<td>40 (h)</td>
</tr>
<tr>
<td>$Y_{+\text{max}}$</td>
<td>30 (h)</td>
</tr>
<tr>
<td>$Y^{-\text{max}}$</td>
<td>30 (h)</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>2 ($/h)</td>
</tr>
<tr>
<td>$F$</td>
<td>60 ($/h)</td>
</tr>
<tr>
<td>$h^P_1$, $h^P_2$</td>
<td>2 ($/unit), 2 ($/unit)</td>
</tr>
<tr>
<td>$p_1, p_2$</td>
<td>10 ($/unit), 10 ($/unit)</td>
</tr>
<tr>
<td>$K$</td>
<td>1.5 ($/unit)</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.5 ($/unit)</td>
</tr>
</tbody>
</table>

### Table 4
Data of model S

<table>
<thead>
<tr>
<th>Constants</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1, p_2$</td>
<td>10 ($/unit), 10 ($/unit)</td>
</tr>
<tr>
<td>$h^S_1$, $h^S_2$</td>
<td>1 ($/unit), 1 ($/unit)</td>
</tr>
<tr>
<td>$s_1, s_2$</td>
<td>1 ($), 1 ($)</td>
</tr>
<tr>
<td>$K^0$</td>
<td>1.5 ($/h)</td>
</tr>
<tr>
<td>$I^1_1, I^1_2$</td>
<td>0 (units), 0 (units)</td>
</tr>
<tr>
<td>$c^1, s^2$</td>
<td>2 (h/unit), 2 (h/unit)</td>
</tr>
<tr>
<td>$C^0$</td>
<td>100 (h)</td>
</tr>
<tr>
<td>$\Delta C^1_{\text{max}}$</td>
<td>80 (h)</td>
</tr>
</tbody>
</table>

### 5.2. Main results of the analysis

#### 5.2.1. Total costs

According to our two questions, let us first analyze the total cost performance of the supply link in relation to the different anticipative coordination types. Fig. 6 gives the corresponding results of a risk analysis. The curves describe (on the basis of 1000 simulation runs) estimates for the probability density of total cost. These costs are obtained in optimizing the appropriate supply link for the specified data (Fig. 5 and Tables 1–5) and in substituting the resulting decisions for each simulated value of $(a_1, \ldots, a_{10})$ into the overall cost criterion (i.e., $(I^1)$ of the ideal model). The optimization is performed in employing the Express MP standard solver and the simulation runs are generated in drawing random numbers from the probability distributions given in Table 5.

Fig. 6 plainly shows the advantage of the reactive anticipation over the remaining types of coordination. It is nearly as good as the (non-realistic) ideal model. In fact, as already stressed in Section 4.2.3, for the reactive anticipation the penalty cost parameter $K$ can be chosen optimally, which is not the case for the pure top-down hierarchy and the non-reactive anticipation. Hence, for a reactive anticipation, penalty costs are not rigid quantities, but rather quantities which reflect the temporary situation of the whole supply link, expressed by the particular series of customer demand. As mentioned before, this flexible behavior of the (reactive) penalty cost is not likely to be met in practice, and Fig. 6 plainly shows the advantage which formally may be seen in the effect of a planning procedure that combines the task-oriented and the control-oriented coupling mechanisms.

Note that for the non-reactive and the pure top-down cases $K$ cannot be optimized, but has to be fixed in advance (Table 3). It can be shown,
however, that, for a large range of reasonably chosen values of $K$, the result depicted in Fig. 6 does not change substantially (see Zimmer, 2001).

Let us now investigate the effect of private information. Let us first consider the situation that the producer is not fully informed about the capacity expansion cost parameter $\jmath$ of the supplier. In a second step we then analyse the effect of private knowledge concerning the supplier’s capacity situation.

5.2.2. Private knowledge about capacity expansion cost

Fig. 7 gives the result for the pure top-down hierarchy, the two different types of anticipation, and the ideal model. Again the comparison is performed in optimizing the different anticipation models and in evaluating total costs (ID1) for these optimal values. More precisely, each point in Fig. 7 represents a mean value of the total cost criterion (ID1) over 500 scenarios of $(\alpha_1, \ldots, \alpha_{10})$. These scenarios were drawn from the probability distributions for $\alpha_t (t = 1, \ldots, 10)$ specified in Table 5.

Starting point of the investigation is a situation in which the supplier is fully informed about the expansion cost parameter $\kappa$. In this case (specified by $\kappa = \kappa^0 = 1.5$) the reactive anticipation is slightly worse than the ideal model and much better than the non-reactive and the pure top-down case (see the results at $\kappa^0$ in Fig. 7).

The actual focus of our investigation, however, is shown for values of $\kappa$ which deviate from $\kappa = \kappa^0$ by a certain percentage. Here one has two effects. First, because of an increased $\kappa$ the solution of the ideal model shows a slight increase in total cost, as it should be. The second effect, and this is what we are really interested in, is the effect of information asymmetry. Assuming that the producer does not know the real value of $\kappa$, she still optimizes her various anticipations in using $\kappa^0$. Hence, the differences between the curves in Fig. 7 do not only describe the quality of the different types of anticipation but also their reaction w.r.t. wrong information. For positive percentage deviations one has with $\kappa^0$ an underestimation of costs whereas for negative deviations one determines the anticipations with a parameter which is overestimated. Note that for the pure top-down hierarchy there is no anticipation, and hence an information asymmetry has no effect, which consequently results in a robust behavior. It is interesting to note that the poorly performing non-reactive anticipation behaves fairly robust as well. The reactive
anticipation, however, shows a considerable sensitivity when $k$ is underestimated by $k_0$. Observe that the cost reduction for an overestimated $k$ ($-50\%$) is due to the high flexibility of the supplier. In this case the supplier only insignificantly restricts the producer. This may readily be drawn from the fact that the pure top-down case, in which the supplier’s role is obsolete, gives almost identical results.

In summarizing, the results depicted in Fig. 7 clearly show that the two anticipations give results that lie between the two benchmarks, i.e., the best case represented by the (non-realistic) ideal model and the worst case described by the pure top-down procedure. The reactive anticipation is always better than the non-reactive one even in case of asymmetric knowledge. However, the reactive anticipation is rather sensitive as to an underestimation of capacity adaptation cost.

5.2.3. Private knowledge about the supplier’s capacity situation

Let us now consider the case that the normal capacity $C$ of the supplier is not known to the producer. (In view of Eq. (S5) or (5), this is equivalent as if the producer is not able to determine correctly an estimate $\hat{z}$ for the mean value of $z$.) Uncertainty w.r.t. the supplier’s normal capacity may represent the effect of sections of the supply net that are not explicitly taken into account. The more the supplier is committed to other producers, the less sure our producer can be to estimate the correct capacity the supplier is allocating for this particular supply link. Obviously, knowledge about the short-term capacity situation may not easily be obtained. Hence, the possibility to gain a correct value for the estimate $\hat{z}$ may be used to characterize the mutual commitment of the two partners in a supply link.

Fig. 8 gives the result. It is constructed according to Fig. 7. Again, the ideal and the pure top-down model represent the best and the worst case, respectively (see the 0% mark in Fig. 8 with $C^0$ specified in Table 4). For the two anticipation models one additionally has the effect of overestimation (negative percentages) and underestimation (positive percentages) of the mean value of $z$. Again the reactive anticipation always outperforms the non-reactive one. Moreover, an overestimation does not effect the performance of the reactive anticipation while an underestimation results in higher costs. (For an explanation of further features of the curves a deeper discussion of the whole structure of the model is necessary which needs additional investigations (see Zimmer, 2001).)
In summarizing one again realizes that using a reactive anticipation results in satisfactory outcomes, provided that the producer does not underestimate normal capacity. Similar results can be obtained for the manufacturing coefficients (capacity consumption rates). For all other parameters, like inventory or setup costs it can be shown that such a crucial dependency on a wrong specification does not exist. Both the non-reactive and the reactive anticipation turn out to be robust w.r.t. misspecifications (see Zimmer, 2001).

6. Summary and concluding remarks

The paper considered operational coordination mechanisms within a supply chain. In describing a particular link within an entire network, we employed main concepts of the theory of hierarchical planning and identified the top-level with the producer and the base-level with the supplier. Both levels were coordinated by an increasing degree of anticipative integration.

For the pure top-down hierarchy, none of the features of the supplier are taken into account, which implies that only a task-oriented instruction is exerted. For the non-reactive anticipation type of integration, important characteristics of the supplier were considered, particularly his capacity situation and the ability to adapt his capacity. This type of anticipation, however, is still on a low level of integration, since it does not account for any reaction to the producer's action. Hence, as in the pure top-down case, one has only a task-oriented type of instruction showing a performance which, essentially, is not much better than the pure top-down case. Only the reactive anticipation which fully takes into account the supplier's model results in a substantially improved outcome deviating only slightly from the ideal situation. Formally, this satisfactory performance is due to the combined effect of a task-oriented and a control-oriented coupling mechanism.

Clearly, the reactive anticipation is not that easy to calculate as compared with the pure top-down and the non-reactive anticipation coordination schemes. In addition, far more information about the supplier needs to be known. On the other hand, the results do not differ too much from the ideal situation for which even more data is necessary and an even more complex analysis must be performed. Moreover, considering the typical situation within a supply chain such a monolithic optimization relying on the traditional one-person paradigm is not of much interest. Hence for a situation in which the parties are willing to closely cooperate, the reactive anticipation can be considered as a reasonable coordination scheme. In
this situation, the only data that is crucial and should be carefully exchanged concerns the capacity situation of the supplier and the costs of a capacity adaptation. For all other parameters a knowledge is not sensitive.

It is interesting to reflect again on the information situation and the team or non-team behavior of the two parties. The situation we considered in this paper seems to us characteristic for SCM and can be characterized by a team with partners still having some privacy. Deviating from this setting, one would arrive at two extreme situations:

1. **Team with no private information.** In this case, employing a reactive anticipation, one has a constructional (i.e. information symmetric) hierarchy (see Schneeweiss, 1999, Chapter 3) and penalty costs are describing a typical coupling term between two separate parts of a large logistic model which has been split up into a producer's and a supplier's component. The whole supply link may be interpreted as a ‘decision generator’, and the ideal model represents the ‘real model’ one is ultimately interested in. Moreover, the penalty cost parameter \( K \) turns out to be a steering cost which is to be optimized \( w.r.t. \) the real model’s criterion (see Schneeweiss, 1999, Chapter 9).

2. **Non-team with private information.** In this second extreme situation one has a principal agent setting (see, e.g., Spremann, 1987), penalty costs turn out to be (negative) incentives, and there are real payments between the parties. An evaluation \( w.r.t. \) the ideal model is no longer reasonable. Rather one optimizes the producer's and supplier's revenues in deriving an appropriate Nash equilibrium of the coupling equations.

The paper primarily focused on the operational performance of a supply chain. It is obvious, however, that before a contract is written, the operational results can be used by the producer in her negotiations with the supplier. In these negotiations one would have to compare information costs with the advantages of a closer coordination which are not only favorable for the producer but for the supplier as well. In particular, in agreeing on a reactive anticipative coordination scheme one would choose, according to our analysis, an optimal value of \( K \) and, in addition, one would try to find an agreement as to how the total contribution margin should be shared; a problem which was not discussed here.

The results are not only of theoretical interest. They provide some hints for practical applications as well. Firstly, they show the effect of situation-dependent penalty cost as opposed to the constant cost term usually encountered in practice. Secondly, one may conjecture that primarily the disclosure of real time capacity data is of crucial significance. Finally, in case of uncertain data, particularly those of real time capacity, an analysis might be helpful showing whether underestimating or overestimating is more critical.

Of course, the present investigation is limited to a particular production situation and to certain selected coordination instruments like order quantity and due date penalty cost. Moreover, the numerical analysis had to be restricted to computationally still tractable models. On the other hand, the models we considered are fairly realistic and, in fact, the logistic structure is more strongly taken into account than it is the case with the highly aggregated models one usually encounters in models of supply chain contracting.

References


