QTOP: A Topological Approach to Minimizing Single-Output Logic Functions

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Abstract— Minimizing Logic functions is of great importance in design and implementation of digital circuits because makes them more efficient and simpler to implement. Therefore, it is considered as an important subject in electrical and computer engineering educational programs. There are some systematic techniques, which are traditionally used in order to teach how to minimize logic functions. These techniques can be easily implemented in the form of computer programs; however, each of them has shortcomings from education point of view. For example, the Quine-McCulski technique is an iterative technique and therefore takes a long time and increases the probability of making mistakes. The Karnaugh map- the other traditional method- causes a visual difficulty in distinguishing adjacent entries and prime implicants. This paper proposes a topological non-iterative approach to minimizing single-output logic functions which is based on representing minterms by nodes in a graph. The main goal of this approach is to represent prime implicants by explicit cycles in graphs in order to eliminate the ambiguity in distinguishing implicants and prevent mistakes.

Keywords: logic function minimization; Karnaugh map; prime implicant.

I. INTRODUCTION

The Quine-McCulski tabulation method \([6,7]\) and the Karnaugh map \([8]\) are traditionally used to teach logic function minimization. These methods can be easily implemented as computer programs \([9,10]\), but each of them has its own educational shortcomings. The Quine-McCulski tabulation method is a time-consuming iterative method and the karnaugh map does not clearly visualize adjacent entries in the map which makes prime implicants difficult to distinguish. There are also methods which have not been developed for educational purposes. Some of these methods rely on mathematical concepts such as cube algebra \([2,3]\) and genetic \([1,5]\) or evolutionary \([4]\) algorithms.

This paper proposes a novel non-iterative method based on topological structures called QTOP (Q\(_n\) TOPOlogy) which eliminates the visual deficiency of Karnaugh maps in demonstrating the adjacency of entries. The main idea behind this method is putting minterms of n logic variables on the nodes of a graph and marking the nodes corresponding to the minterms of the function to be simplified. A Q\(_n\) graph can be recursively defined as follows.

A Q\(_1\) graph consists of 2 nodes and a single edge connecting the two nodes.

A Q\(_n\) graph consists of two Q\(_{n-1}\) graphs whose corresponding nodes have been connected by new nodes. Figure 1 shows a Q\(_1\) graph, and figure 2 shows two topologically equivalent forms of a Q\(_2\) graph and figure 3 demonstrates three equivalent forms of a Q\(_3\) graph.
that each node in a $Q_n$ graph is of degree $n$. That is there are $n$ edges connected to each node. The latter property allows minterms that are different in exactly one literal to be represented by adjacent nodes in a $Q_n$ graph. Due to these properties, each cycle consisting $2^n$ marked nodes represents a prime implicant of the corresponding function. As a simple example, consider the 3-variable function $f(x, y, z)$ as follows.

$$f(x, y, z) = \sum (0, 2, 4, 6)$$

$$= x'y'z' + x'yz' + xy'z' + xyz'$$

Equation (1)

Figure 3 shows the Karnaugh map of the above function.

The 4 minterms of the function $f(x, y, z)$ construct a 4-prime implicant but as shown in figure 4, The Karnaugh map is unable to represent the implicant by a visually contiguous area. But the same implicant can be visually distinguished if $f(x, y, z)$ is represented by a $Q_3$ graph as demonstrated in figure 5.

As shown in figure 5, The QTOP diagram clearly shows the 4-prime implicant as a 4-cycle. This method can be generalized for minimizing logic functions with more input variables.

REFERENCES


