Shared-PPRM: A Memory-Efficient Representation for Boolean Reversible Functions

Yasaman Sanaee^a, Mehdi Saeedi^b, Morteza Saheb Zamani^b

Quantum Design Automation Lab

Amirkabir University of Technology, Computer Engineering Department, Tehran, Iran

Email: ^aysanaee@dena.aut.ac.ir, ^b{msaeedi,szamani}@aut.ac.ir

Abstract

A memory-efficient representation scheme, shared-PPRM (SPPRM), for Boolean reversible functions is introduced and analyzed. Compared with conventional PPRM expansion, SPPRM reduces memory usages by using one memory location for many repetitive PPRM sub-expressions. To evaluate the effects of data structure on SPPRM representation, two linked list-based data structures are also examined. The experimental results show the efficiency of the proposed SPPRM representation for both memory usage and CPU time.

1. Introduction

Positive polarity Reed-Muller expansion (PPRM) has been used to represent reversible functions extensively. However, its memory usage could be too high which limits the applicability of algorithms. On the other hand, as the size of a reversible function increases, its truth table size grows exponentially. Therefore, an efficient data structure to reduce memory usages is vital.

2. SPPRM Representation

Since each n -variable reversible function F has n output variables, i.e. $f_1, f_2, \dots, f_n$, the PPRM expansion of F contains n PPRM expressions each of which is the result of EXORing several, from 1 to 2^n , sub-expressions.

Definition 1: A given n -input n -output Boolean reversible function can be represented by a *shared PPRM (SPPRM)* representation if its PPRM expressions have common sub-expressions. In such a situation, the common sub-expressions are represented for all n expressions only once.

Definition 2: Given an n -input, n -output Boolean reversible specification F represented as Equation (1) with input variables $x_1, x_2, \dots, x_n$, the memory cost for

the (S)PPRM representation of F is defined as Equation (2) where $a_{i,j} \in \{0, 1\}$.

$$\begin{aligned} f_1 &= a_{1,1} \oplus \dots \oplus a_{1,2^n} x_1 x_2 \dots x_n \\ f_2 &= a_{2,1} \oplus \dots \oplus a_{2,2^n} x_1 x_2 \dots x_n \\ &\dots \\ f_n &= a_{n,1} \oplus \dots \oplus a_{n,2^n} x_1 x_2 \dots x_n \end{aligned} \quad (1)$$

$$MC = \sum_{i=1}^n \sum_{j=1}^{2^n} a_{i,j} \quad (2)$$

Theorem 1: Suppose an n -input n -output reversible specification F . Assume that an m -term ($m \leq 2^n$) sub-expression v is common among k ($k \leq n$) output variables. Converting the PPRM expansion of F to SPPRM expansion improves the memory cost if the following equation is held:

$$m > \frac{k}{k-1} \quad (3)$$

3. The Effects of Data Structures

To use SPPRM expansion, an efficient data structure is needed. In this section, two linked list-based data structures are introduced and analyzed. For each sub-expression of a PPRM expansion, a distinct linked list is considered as shown in the following example.

Example 1: Consider a given specification $F = (3, 4, 6, 7, 0, 1, 5, 2)$. Fig. 1 illustrates the linked list representation of the PPRM expansion.

Fig. 1 Linked list-based data structure for PPRM expansion of Example (1)

In order to use the linked list-based data structure for SPPRM representation, one may use a separate linked list for each common sub-expression and use its memory address for all related output variables as demonstrated in Fig. 2.

Fig. 2 Type-1 linked list-based data structure for SPPRM expansion of Example (1)

On the other hand, it may be possible to represent the common sub-expression for only one output variable with no internal node requirement. Other related output variables should refer it as shown in Fig. 3.

Fig. 3 Type-2 linked list-based data structure for SPPRM expansion of Example (1)

It can be verified that Type-2 data structure is more efficient than Type-1 in terms of memory usage. Therefore, for the rest of this paper, Type-2 data structure is used to evaluate the efficiency of SPPRM representation.

While SPPRM could reduce memory usage inherently, SPPRM construction has a negligible timing overhead. In this section, a PPRM-to-SPPRM

conversion algorithm is described. Using Type-2 data structure, the input of conversion algorithm is n linked lists each of which contains some sub-expressions. The output of the algorithm is the resulted SPPRM expansion. Fig. 4 shows the SPPRM extraction algorithm. As illustrated in this figure, for each m and k values (see Theorem 1), all combinations of k output variables are found and used to select the possible common sub-expression. In other words, for each combination, a common sub-expression is chosen and its memory usage improvement after SPPRM representation is found. After finding all possible common sub-expressions, the best one with the maximum memory usage improvement is selected and used to construct SPPRM expansion. The time complexity of the algorithm is $O(2^{2n})$.

4. Experimental Results

The proposed SPPRM expansion and the applied data structure were implemented in C++ and all of the experiments were done on an Intel Pentium IV 3 GHz computer with 1GB memory. To evaluate the efficiency of the SPPRM expansion, all 40320 3-input 3-output reversible Boolean functions were examined. In other words, the numbers of functions with common sub-expressions in their 2 or 3 output variables were calculated. In addition, the percentages of memory improvement after using SPPRM representation were extracted. The results of using SPPRM expansion for 3-input 3-output reversible functions are shown in Table 1. It can be verified that more than 16% improvements are obtained in memory usage with no considerable CPU overhead.

In addition, the results of using SPPRM expansion for 16 examples taken from the literature [4] were also shown in Table 2. As shown in this table, 18.5% improvement is achieved on average.

Algorithm SPPRM Extraction

Input: n PPRM expressions $f_1, f_2, \dots, f_n$ in a linked list data structure

Output: n SPPRM expressions

1. for $k=2$ to n do
2. select the values of m based on the results of Theorem 1
3. select all possible k output variables
4. find common sub-expressions among selected output variables
5. If total number of terms in common sub-expressions is greater than m then
6. insert the common sub-expression into common_list
7. end
8. end
9. select the best possible common sub-expression from common_list with the maximum memory usage improvement
10. construct memory locations for common sub-expression
11. update output variable linked lists

Fig. 4 SPPRM extraction algorithm

Table 1- SPPRM Memory improvement for all 3-input 3-output reversible functions

# of sub-expressions	# of functions		# of sub-expressions	# of functions	
	PPRM	SPPRM		PPRM	SPPRM
1	0	0	10	6426	9024
2	0	0	11	7938	5220
3	6	6	12	7440	2148
4	72	108	13	5004	0
5	324	612	14	2250	0
6	870	1860	15	690	0
7	1782	4446	16	180	0
8	2916	7308	17	36	0
9	4380	9588	18	6	0

Table 2- SPPRM expansion for several examples [4]

ckt #	Specification	Memory usage		
		PPRM	SPPRM	Imp%
1	(1,0,3,2,5,7,4,6)	8	8	0
2	(7,0,1,2,3,4,5,6)	10	6	40
3	(0,1,2,3,4,6,5,7)	7	7	0
4	(0,1,2,4,3,5,6,7)	13	7	46.15
5	(0,1,2,3,4,5,6,8,7,9,10,11,12,13,14,15)	34	13	61.76
6	(1,2,3,4,5,6,7,0)	6	6	0
7	(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,0)	8	8	0
8	(0,7,6,9,4,11,10,13,8,15,14,1,12,3,2,5)	10	10	0
9	(3,6,2,5,7,1,0,4)	11	11	0
10	(1,2,7,5,6,3,0,4)	11	8	27.3
11	(4,3,0,2,7,5,6,1)	13	9	30.8
12	(7,5,2,4,6,1,0,3)	12	9	25
13	(6,2,14,13,3,11,10,7,0,5,8,1,15,12,4,9)	24	21	12.5
14	(9,7,13,10,4,2,14,3,0,12,6,8,15,11,1,5)	26	21	19.2
15	(6,4,11,0,9,8,12,2,15,5,3,7,10,13,14,1)	29	24	17.2
16	(13,1,14,0,9,2,15,6,12,8,11,3,4,5,7,10)	24	20	16.66
average				18.53

To further evaluate the cost of using SPPRM expansion, we examined 33 reversible benchmarks [6] the percentages of memory improvements of which are

shown in Table 3. As shown in Table 3, 9.72% improvement is achieved on average.

Table 3- The results of using SPPRM expansion for reversible benchmarks

circuit #	Benchmark	Memory usage		
		PPRM	SPPRM	Imp%
1	2of5	45	32	22.2
2	3of17	14	10	28.8
3	4mod5	15	12	20
4	4of49	39	30	23
5	5mod5	25	25	0
6	5one013	67	46	31.3
7	5one245	111	86	22.5
8	6one0246	12	12	0
9	6one0135	11	11	0
10	alu	55	49	10.9
11	decod24	30	24	20
12	graycode10	19	19	0
13	graycode20	39	39	0
14	graycode6	11	11	0
15	ham3	8	4	50
16	ham7	32	29	9.8
17	hwb4	28	22	21.4
18	hwb5	85	75	11.7
19	majority	9	9	0
20	majority5	55	46	16.7
21	majority7	289	259	10.4
22	mod15adder	23	23	0
23	mod32adder	41	41	0
24	mod5adder	82	82	0
25	mod64adder	75	75	0
26	rd32	9	9	0
27	rd53	30	30	0
28	rd73	104	104	0
29	shift10	43	43	0
30	shift115	59	59	0
31	shift28	111	111	0
32	xor5	9	9	0
33	2of5	45	32	22.2
average				9.72

To evaluate the scalability of the proposed SPPRM expansion, several other experiments were also performed. We generated 500 random circuits of different input sizes. Then, both PPRM and SPPRM expansions were constructed. The numbers of memory usages are reported in Table 4. As shown in Table 4, using SPPRM for reversible circuit representation leads to better memory usage with a negligible CPU overhead.

5. Conclusion

Today, reversible logic has received considerable attention in various research areas. In this paper, a memory-efficient representation scheme, SPPRM, for Boolean reversible functions was introduced and evaluated. In addition, the effects of various data structures on its efficiency were examined. Compared with the commonly used PPRM expansion, SPPRM could lead to a great improvement on memory usage.

6. References

- [1] R. Landauer, "Irreversibility and Heat Generation in the Computing Process," IBM Journal, vol. 5, pp. 183-191, July 1961.
- [2] C. Bennett, "Logical Reversibility of Computation," IBM Journal, vol. 17(6), pp. 525-532, November 1973.
- [3] A. K. Prasad, V. V. Shende, I. L. Markov, J. P. Hayes, K. N. Patel, "Data Structures and Algorithms for Simplifying Reversible Circuits," ACM Journal on Emerging Technologies in Computing Systems, Vol. 2, Issue 4, 2006.
- [4] M. Saeedi, M. Saheb Zamani, M. Sedighi, "On the Behavior of Substitution-Based Reversible Circuit Synthesis Algorithms: Investigation and Improvement," International Symposium on VLSI, 2007.
- [5] P. Gupta, A. Agrawal, and N. K Jha, "An Algorithm for Synthesis of Reversible Logic Circuits," IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, Vol. 25, Issue 11, pp. 2317-2330, 2006.
- [6] D. Maslov, "Reversible Logic Synthesis," Ph.D. dissertation, The University of New Brunswick, 2003.

Table 4- The results of using SPPRM expansion for random circuits of various sizes

# of sub-expressions	# of functions									
	4-input		5-input		6-input		7-input		8-input	
	(P)	(SP)	(P)	(SP)	(P)	(SP)	(P)	(SP)	(P)	(SP)
[min, min+k)	4	1	7	5	4	2	3	4	4	11
[min+k, min+2k)	4	7	12	8	23	13	3	5	22	25
[min+2k, min+3k)	21	0	47	48	62	26	19	24	56	64
[min+3k, min+4k)	37	38	71	55	119	87	44	71	106	107
[min+4k, min+5k)	94	30	101	62	116	92	95	100	101	102
[min+5k, min+6k)	103	130	93	133	93	133	97	114	123	101
[min+6k, min+7k)	112	144	103	85	52	83	100	96	58	57
[min+7k, min+8k)	79	65	42	68	26	38	89	57	20	18
[min+8k, min+9k)	31	70	14	25	4	22	35	26	9	14
[min+9k, max]	15	15	10	11	1	4	15	3	1	1
min	18	14	61	53	163	143	394	362	955	1097
max	39	30	94	77	222	189	479	435	890	1001
CPU overhead(sec)	0.03		0.12		0.52		2.09		9.09	
									40.34	

- P and SP have been used as PPRM and SPPRM, respectively.
- $k = (\max - \min) / 10$
- 500 random circuits were generated for a given number of inputs